The Neoclassical Growth Model and Twentieth-Century Economics

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While growth has been a central element of economic thought at least since the physiocrats and Adam Smith, the modern analysis of growth using formal models began only in the middle of the twentieth century. Thanks largely to Robert Solow's two articles, "A Contribution to the Theory of Economic Growth" (1956) and "Technical Change and the Aggregate Production Function" (1957), growth economics developed into a major area of research in macroeconomics and economic theory, attracting the attention of a significant part of the economics profession.

The current volume collects most of the papers from the twentieth annual *HOPE* conference, held 25–27 April 2008 at Duke University. The conference addressed the history of modern growth economics, taking Solow's key papers from the 1950s as its anchor. The conference was not about Solow's work per se, but addressed the intellectual currents that formed the background to that work and the history of the growth economics that it subsequently informed. The conference considered the rise of growth economics as an active field of research in the 1950s, its extension to several branches of the discipline in the 1960s, its decline in the 1970s, and its return to the center stage of macroeconomics over the last twenty years. In addition to sixteen essays presented at the conference, we are fortunate to be able to include transcripts of two less formal talks. The first is Professor Solow's keynote lecture to the conference on the future of growth economics. This lecture was delivered as part of a celebration

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of Professor Solow's commitment of his papers to the Economists' Papers Project of Duke University's Rare Book, Manuscript, and Special Collections Library. The second is Edwin Burmeister's recollections of his time as Solow's student in the graduate program in economics at the Massachusetts Institute of Technology (MIT)—an after-dinner talk on the first night of the conference.

Although it was a relatively minor field in the early days of macroeconomics, growth economics exploded in the decade and a half after Solow's 1956 paper (see table 1). Through the 1960s the basic neoclassical growth model was extended in several directions, by Hirofumi Uzawa (two-sector model), Kenneth Arrow (learning by doing), James Tobin (money and growth), Peter Diamond (fiscal policy and overlapping generations), and many others. Edward Denison, Zvi Griliches, and Dale Jorgenson, among others, elaborated Solow's (1957) approach to growth accounting. Solow himself modified the simple model to introduce the notion of vintage capital with embodied technological change and worked out a new version without direct substitution between factors of production. During that same period, Edmund Phelps and others used Solow's model—now frequently known simply as the *neoclassical growth model*—to establish the golden rule of growth, while David Cass and others combined it with Frank Ramsey's much older model of capital accumulation to study optimal growth. At the same time, the growth models of Roy Harrod, Evsey Domar, and John von Neumann continued to attract some (declining) attention, while alternative approaches to growth theory (especially those of Nicholas Kaldor and other Cambridge economists) were still very much on the agenda. F. H. Hahn and R. C. O. Matthews's famous survey of growth economics, published in the *Economic Journal* in 1964, summed up the main results and stabilized the discussion for some time.

Under the influence of Solow and Paul Samuelson, MIT quickly became the main center of research in growth theory, with several PhD students rising to prominence as growth theorists, including Diamond, Eytan Sheshinski, William Nordhaus, Burmeister, Joseph Stiglitz, A. Rodney Dobell, and Avinash Dixit (see the Festschrift edited by Diamond in celebration of Solow's sixty-fifth birthday). In 1965–66 Karl Shell organized an influential seminar on optimal economic growth at MIT, which resulted in a conference volume published the next year. By 1970 economists started to take stock, and growth theory began to be consolidated in textbooks, including those of Burmeister and Dobell (1970) and Henry

Table 1 Attention Paid to Growth by Economists

Period	Articles with "growth" in title (as percentage of all articles)	Change from previous period (percent)
1936–55	0.95	
1956–70	4.64	388
1971–85	2.71	–42
1986–2006	2.67	–2

Data derived from JSTOR journal archive (30 October 2006) based on the *American Economic Review*, the *Journal of Political Economy*, the *Review of Economic Studies*, the *Review of Economics and Statistics*, the *Economic Journal*, and *Econometrica*.

Wan (1971), also an MIT PhD. Solow's 1969 Radcliffe Lectures (published in 1970 as the first edition of his *Growth Theory: An Exposition*) became a standard reference, along with two collections of readings in growth economics—one edited by Amartya Sen (1970), the other by Stiglitz and Uzawa (1969).

Interest in the theory of economic growth subsided in the 1970s and early 1980s, when only a few new results were produced, such as the application of the neoclassical growth model to the economics of exhaustible resources by Solow and others in the wake of the surge in the price of oil following the Yom Kippur War of 1973. Although the main focus of macroeconomic research shifted in this period back toward short-term fluctuations, table 1 shows that interest in growth did not fall back to its pre-1956 level. The middle 1980s seemed to provide a new beginning for the economics of growth. Even though interest in growth measured by the crude statistics of table 1 was merely steady, Paul Romer and Robert Lucas opened up a new research agenda, one that has persisted for the last two decades (see also Kim, Morse, and Zingales 2006).

The *new growth theory* extended the neoclassical model by treating the steady-state rate of growth as itself endogenous, in the sense that it is affected by taste parameters (such as the savings rate) and/or is determined within the model. At first, an endogenous growth rate was secured by replacing Solow's assumption of diminishing returns to capital by constant returns to capital broadly defined. A second phase focused on monopolistically competitive models in which the rate of technological progress was endogenous (see David Warsh's 2006 fascinating historical account, focused on Romer's contributions).

Solow engaged in the debate, and he reacted critically to the theoretical and empirical aspects of new growth economics in his 1992 Siena Lectures (revised and incorporated into the second edition of his *Growth Theory*, published in 2000), in his contributions to the *Handbook of Macroeconomics* in 1999 and to the *Handbook of Economic Growth* in 2005, and elsewhere. By the mid-1990s textbooks started to discuss endogenous growth models extensively (Barro and Sala-i-Martin 1995; Jones 1998; Aghion and Howitt 1998), yet they continued to take the basic neoclassical growth model as their common starting point.

I

Solow (1956) set out an aggregative, competitive general equilibrium perfect-foresight growth model built around three equations: a constant-returns-to-scale production function with smooth substitution and diminishing returns to capital and labor; an equation describing capital accumulation on the assumption of a constant rate of savings (investment) as a fraction of output; and a labor-supply function in which labor (population) grows at an exogenously given rate.[1] The system generated a first-order differential equation that showed how the current level of the capital-labor ratio and two parameters (the savings rate and the rate of population growth) determine the rate of change of the capital-labor ratio. Solow provided quantitative solutions for specific constant-returns production functions (such as the Cobb-Douglas and CES functions). Solow first analyzed the dynamic stability of equilibrium qualitatively using a diagram—known as the "Solow diagram"—depicting the equilibrium value of the capital-labor ratio in the steady state. The diagram showed how the economy would converge to a steady-state growth path along which output and the capital stock both grew at the exogenous rate of population growth. To account for increasing income per capita, Solow briefly introduced technical change and worked out the solution for the Cobb-Douglas case, with the result that, along the balanced growth path, output per worker and capital per worker both grow at the same rate of exogenous technologi-

1. What Solow and others characterize as the savings function would more naturally be thought of as the investment function. The identification works only because Solow assumes continuous full employment and the equality of *ex ante* and *ex post* savings and investment rates, which guarantee that both planned and realized investment and savings are equal.

cal progress.[2] Technological improvement, therefore, offsets diminishing returns to capital accumulation, permitting steadily rising labor productivity and output per worker.

While Karl Marx, Knut Wicksell, and Alfred Marshall may have known the notion of steady-state growth in an incipient form already in the nineteenth century, it was Gustav Cassel in the early twentieth century who elaborated the idea and introduced it into the literature under the guise of "the uniformly progressing state." Mauro Boianovsky's essay in this volume documents the influence of Cassel's growth model on other Swedish economists, especially Erik Lundberg, who shared with Harrod's and Domar's later formulations not just the growth equations but also (differently from Cassel) their implications for economic instability. Cassel's and Lundberg's contributions were not themselves enough to set off growth economics as a main field of research worldwide. Things started to change in the late 1940s, not just because of the publication of Harrod's book (*Towards a Dynamic Economics* [1948]) and Domar's articles ("Capital Expansion, Rate of Growth, and Employment" [1946] and "Expansion and Employment" [1947]) but also because of the general concern with economic development and growth after the Second World War.[3]

At the outset of his 1956 paper, Solow claims that he was reacting to what he described as the "knife-edge" property of the "Harrod-Domar model." Solow sees this knife-edge property as the result of an inconsistency between Harrod's *warranted rate* of growth, determined as the ratio of the savings rate and the capital-output ratio, and his *natural rate* of growth, determined by the rates of technical progress and population growth. Solow argues that Harrod's and Domar's models assume that the capital-output ratio is rigidly determined by a fixed-coefficients production function and, therefore, that no mechanism exists to bring the warranted and natural rates into line. He proposes that a flexible production function with substitutability between capital and labor will provide a mechanism for establishing the equality between the two rates and so

2. Technical progress in the form Solow employs later became known as *output-augmenting* or *Hicks-neutral* technical progress. A few years later, Hirofumi Uzawa (1961) established that only *labor-augmenting* or *Harrod-neutral* technical progress is compatible with steady-state growth (see also Solow [1970] 2000, 30–35). However, the Cobb-Douglas production function is a special case in which Hicks-neutral and Harrod-neutral technical progress are equivalent.

3. Harrod's 1939 essay went relatively unnoticed until the late 1940s.

will eliminate the knife-edge. In his essay in this volume, Harald Hagemann shows that Solow's framework was quite distinct from Domar's and Harrod's. In particular, Solow excluded from his model the possibility of divergence between Harrod's warranted (or equilibrium) and actual rates of growth. Harrod relied on such divergences and the resulting instability of the warranted rate, itself caused by discrepancies between savings and planned investment in his complex interpretation of cyclical fluctuations. As Hahn and Matthews (1964) and Sen (1970, introduction) point out, Solow considered only one aspect of Harrod's instability problem (the balance between warranted and natural growth). He neglected the balance between warranted and actual growth, which Harrod related to entrepreneurial expectations.

Solow wrote to Sen (14 January 1970) that he agreed with the introduction to Sen's anthology. He acknowledged that "it is clear to me that my general discussion in the 1956 article was ambiguous, for the simple reason that it wasn't clear to me at the time exactly what I was doing."[4] In another letter, Solow acknowledged that the object of the 1956 model was to trace full-employment (not actual) paths toward the steady state (quoted in Sen 1970, 24 n. 15; see also Solow [1970] 2000, 21). Solow returned to the topic in his 1987 Nobel lecture, in which he referred to his "youthful confusion" between Harrod's two instability problems (Solow [1970] 2000, xiv).

By the mid-1960s, Solow (1966) had resolved the confusion in his own mind:

> It is clear to me that I oversimplified matters in 1956. The model was new and I didn't understand all its implications. Some of what Harrod called instability is, of course, a matter of the behavior of effective demand, off equilibrium paths. Harrod never specified very clearly what he had in mind, and indeed there is very little in the literature even now that marries the theory of growth and effective demand. What I was getting at in 1956 was this: the special character of Harrod's model rests in the fact that the natural and warranted rates of growth are independent numbers. . . . That characteristic of the model rests on fixed

4. In describing the balance between warranted and natural growth in Harrod's model as a fluke, Solow was in good company: Joan Robinson (1956, 404–6) and other Cambridge economists held similar views, although their solution in terms of the adjustment of saving propensity caused by changes in income distribution along the growth path was quite distinct from Solow's (see Kregel 1980).

> proportions. (It is immaterial whether Harrod believed that factor proportions are technically fixed or simply never change.) In turn, at least some aspects of "instability" arise because the economy is always being pulled away from the warranted path because it differs from the natural path.

Solow (1956, 91), therefore, maintained his earlier position that aggregate disequilibrium should not be ascribed to any deviation from a narrow balance between warranted and natural growth, though it might arise from the "difficulties and rigidities" highlighted by Keynesian short-run analysis.[5] After Kaldor (1961) put forward his famous "stylized facts" of long-run growth, Solow reaffirmed the ability of the neoclassical growth model to account for the main empirical features of steady growth as opposed to the sharp instability results of the Harrod-Domar model, especially if another "stylized fact" is added to the list—the fluctuation of the unemployment rate within fairly narrow limits in modern industrial economies (Solow [1970] 2000, chap. 1, esp. 28).

Domar reacted positively to Solow's aggregative neoclassical growth model (Hagemann, this volume; Easterly 2001, 28). Domar (1957, 7–8) later noted that before the Solow 1956 paper, he had treated capital as the only explicit production factor because he thought including labor as well would require a complex, highly disaggregated production function along the lines of Wassily Leontief's dynamic input-output system. Indeed, Domar (1947, 45) had already referred to Tinbergen's (1942) neoclassical treatment, which he did not follow on the grounds that he wanted to "express the idea of growth in the simplest possible manner."

In contrast, Harrod (1960) denied Solow's interpretation. He claimed that he had not omitted the influence of the rate of interest on the capital-output ratio, but he also acknowledged that he had not paid enough attention to that effect in the long run. To "remedy this defect" he developed—under the partial influence of Ramsey—the notion of a "natural rate of interest" and of an "optimum rate of saving" corresponding to the natural rate of growth, which was the object of correspondence with Solow and led to a further restatement by Harrod (1963, 404–10).

5. Solow fails to acknowledge Harrod's focus is directed exactly to the Keynesian short run. Harrod's dynamic problem is not Solow's long-term growth problem. For a more detailed contrast between Solow's and Harrod's approaches, see Hoover 2008, which also investigates the empirical performance of Harrod's model in accounting for growth fluctuations in the American economy in the twentieth century.

Whereas some commentators have identified the neglect of the business cycle and the ensuing lack of integration between short-run and long-run macroeconomics as the main shortcoming of neoclassical growth theory, Lionello Punzo argues in his essay in this volume that Solow's model was born precisely from the failure of the research program of "classical macrodynamics" to find a unified explanation for cycles and growth. That is, by separating growth and fluctuations, Solow succeeded where other macroeconomists—including not only Harrod and Domar but also Ragnar Frisch, John Hicks, and Richard Goodwin, among others—had failed before him. That "success," however, was achieved by changing the question. While Harrod considered disequilibrium dynamics, neoclassical growth theory considered only equilibrium dynamics. Punzo maintains that the origins and relative success of neoclassical growth theory should be sought less in the influence of the general equilibrium theory and Samuelsonian stability analysis than in its ability to forge a new concept of a dynamics driven by exogenous forces rather than endogenous fluctuations.

As Solow points out in his keynote address to the *HOPE* conference, the development of new theories and approaches in economics is more the result of the collective effort of research communities than of single individuals. Although Solow's contributions have been particularly influential, several aspects of the neoclassical growth model were advanced somewhat earlier or simultaneously with Solow 1956 by Jan Tinbergen ([1942] 1959), James Tobin (1955), and especially the Australian economist Trevor Swan (1956).

In their essay in this volume, Robert Dimand and Barbara Spencer discuss the similarities and differences between Swan's and Solow's presentations of the same basic growth model and assess why, despite the fact that it is often rightly called the "Solow-Swan model," it was Solow's version that caught the eyes of the profession.[6] Both Solow and Swan established mathematically and diagrammatically how the economy finds the steady-state growth path in a one-commodity world. Instead of a general constant-returns production function, Swan worked out the mathematics of growth under the assumption of a Cobb-Douglas function. What is more, it is the capital-output ratio rather than Solow's capital-labor ratio that takes pride of place in Swan's diagram.[7]

6. Barbara Swan Spencer is Trevor Swan's daughter.

7. See also Harcourt's (2006, 110–13) detailed discussion of Swan's diagram. According to Harcourt, "Generations of Australians have been brought up on this and other famous Swan diagrams."

Whereas Solow reacted to what he perceived as the “knife-edge” property of the Harrod-Domar model, Swan sought to contrast the role of capital accumulation in the classical (Ricardian) and neoclassical frameworks, and to criticize Joan Robinson’s (1956) views about growth and capital. With that target in mind, Swan used his model to clarify the role of land in classical growth theory (an issue set aside by Solow 1956) and, more important, to show for the first time that in the neoclassical approach with diminishing returns to capital and exogenous population growth a higher savings propensity brings about only a temporary, not a steady-state, increase of the rate of output growth (an implication of Solow’s model, but not one stressed in his original paper). Swan saw a contrast between the neoclassical growth model, in which the steady-state growth rate is independent of the savings rate (a property that he notes generally vanishes, even in the neoclassical model, if the rate of population growth is endogenously determined by income per capita), and both the classical growth economics and the Harrod-Domar model as it was typically applied to planning for economic development (Easterly 2001). Harrod and Domar themselves were much more careful about the implications of higher savings for economic growth than were the development economists. And, of course, *classical* growth models typically assume endogenous labor-force growth in the form of Malthus’s population mechanism. Swan confirms a well-known classical proposition: the combination of diminishing returns to land and Malthusian population dynamics in the absence of technical progress results in an equilibrium with declining output per head that converges to a stationary state in which population does not grow and capital does not accumulate. Swan’s independence proposition soon became recognized as one of the main implications of diminishing returns to capital in the neoclassical growth model (see Meade 1961, 42–46; Solow [1970] 2000, 22–24).

As Steven Durlauf observed at the *HOPE* conference, despite the striking similarities of their theoretical models, Solow’s research program was empirical in a sense that Swan’s was not. Indeed, Solow followed up his 1956 paper with the equally influential empirical study of the sources of U.S. economic growth between 1909 and 1949 (Solow 1957). The 1957 paper does not refer directly to the 1956 paper, yet the connection between the two is clear enough. Both model the economy in which a neoclassical aggregate production function governs output and in which each factor price equals its marginal productivity. These assumptions allowed Solow (1957) to parameterize the production function.

Solow decomposed the growth of output into the sum of the capital and labor inputs, each weighted by their shares in national income, plus a term that captured Hicks-neutral shifts of the production function. Although he dubbed such shifts "technical change," he acknowledged that they comprised not just technological progress in the narrow sense but also any other changes that affected the productivity of inputs, such as increasing skills among workers. Domar (1961) soon described the contribution of technical change broadly defined to the growth of output per head as the "residual," since it was obtained after calculating the contribution of capital. (The residual is also often called "total factor productivity" [TFP], a term introduced by John Kendrick [1956].) Solow concluded from his 1957 study that about seven-eighths of the increase in output per head in the American economy was traceable to technical change.[8] That was a surprising result, since, although the irrelevance of the investment rate for steady-state growth was clear enough from the 1956 model, its measured small effect even in the transition to the steady state was unexpected.

The massive importance of "technical change" in explaining growth, the exogenous and disembodied manner in which it enters the production function, and the ambiguity of its interpretation (what are its sources? how does it work?) encouraged Solow to investigate further. Solow (1960) offers a model of the accumulation of vintage capital with embodied technical progress. In this model, investment is the vehicle that transmits new ideas. Solow's new approach was the culmination of a line of research that Ingvar Svennilson had begun in Sweden in the mid-1940s (see Boianovsky, this volume).

Domar (1961), among others, noted the clear methodological difference between Solow's aggregative approach to technical change and the disaggregated dynamic input-output approach of Leontief et al. (1953). Marcel Boumans (this volume) argues that Solow's representation of technical change evolved from his work in the early 1950s (frequently with Samuelson) on the stability of dynamic linear systems. Tinbergen advanced a similar representation of technical change in an article originally published in German in 1942 (translated into English in 1959).[9] Indeed, Tinbergen's approach to growth accounting was also based on the notion of

8. According to Warsh (2006, chap. 11), Solow's theoretical and empirical conclusions about the pivotal role of technological progress in the growth process were significant at the time in the context of the Cold War.

9. Although, as he informed the *HOPE* conference, he could read German, Solow did not know of Tinbergen's article at the time he formulated his own growth model.

the residual ("efficiency"), measured with the aid of a neoclassical (Cobb-Douglas) production function.[10] Tinbergen's model was based on the explicit separation between trend and fluctuations. He referred to Cassel, but not to Harrod. The model comprises three equations: a Cobb-Douglas function, which shifts over time; an equation governing equilibrium in the labor market; and an equation governing capital accumulation through proportional savings. Tinbergen assumed numerical values for the parameters of the production function (based on Paul Douglas's previous studies) and solved for the rate of growth of capital. His main concern was to estimate the factors determining long-run growth, without a close theoretical investigation of the steady-state path.

II

In the final pages of his 1956 article, Solow mentions various "cobwebs," including uncertainty and price rigidity, as potentially practically important but foreign to the central message of his model. During the process of spread and consolidation, later neoclassical growth economists have typically followed Solow's lead and neglected to clear the cobwebs—or even to investigate them closely. William Darity (this volume) claims that once (Keynesian) uncertainty is incorporated, the structure of the model itself is affected by the expectations of economic agents, in a manner reminiscent of Harrod's growth dynamics. Darity, a former student of Solow's, adds another cobweb to Solow's list: increasing returns to scale, which would render orthodox marginalist distribution theory useless. Piero Sraffa's (1926) discussion of returns to scale more than eighty years ago set off a debate that led to monopolistic competition models, which have been applied to growth economics by Romer and others since the 1980s.[11]

In contrast, Darity develops another path from Sraffa's criticism of neoclassical theory to a reformed model of growth: the classical surplus

10. Before Tinbergen, Cassel (1935, chap. 6) had considered the empirical extension of the Cobb-Douglas function to the aggregate growing economy. He rejected the idea on the grounds that it would imply a constant capital-labor ratio in a growing economy, which data would not support. That is, Cassel did not take into account the possibility of introducing steady technological progress into the aggregate production function, as Tinbergen and Solow would do later.

11. Solow (1956, 79 n. 7) considered replacing perfect competition with monopolist competition in his model. He gave it up because of the analytic problems involved in its introduction into aggregative models. The first general equilibrium models of monopolist competition were advanced in the 1970s by Avinash Dixit and Joseph Stiglitz (1977) and others.

approach to distribution and growth, which was intertwined with the famous "two Cambridges" controversy in capital theory of the 1950s and 1960s. In his talk (reproduced in this volume), Ed Burmeister, who participated actively in those debates, offers a vivid recollection of that intense discussion from the MIT (Cambridge, Massachusetts) point of view.[12]

Another important development in the 1960s was the shift in focus of growth economics from production to consumption and saving. David Cass and Tjalling Koopmans combined Frank Ramsey's 1928 model of optimal saving with the golden rule of capital accumulation, originally proposed by Edmund Phelps and others in the context of a Solow-Swan model, to create a theory of optimal growth.[13] Although it was initially interpreted as a study of the normative implications of neoclassical growth models, in the hands of Miguel Sidrauski (1967) and especially William Brock (1974) and others in the 1970s (see McCallum 1996, 49) optimal growth gradually came to be seen as offering a positive theory of the actual accumulation path of the market economy as described by the behavior of a representative economic agent with perfect foresight—a transformation that Solow ([1970] 2000, 109) criticized. In comparison with the original Solow-Swan model, the optimal version of the neoclassical growth model yields the same steady-state growth rate, but typically a different steady-state rate of output per worker, as well as different paths away from the steady state.

Had Ramsey's model of optimal savings been ignored before the arrival of the optimal growth literature? In his essay in this volume, Pedro Garcia Duarte argues that Ramsey's 1928 article was read and appreciated by a significant number of economists even before the 1950s, but that it was only after the neoclassical synthesis that dynamic welfare economics and, therefore, Ramsey's approach (as rediscovered and transformed by Cass and Koopmans) were fully integrated into economic theory.

The Solow-Swan growth model abstracted from monetary problems. In their contribution to this volume, Robert Dimand and Steven Durlauf trace the introduction of money into neoclassical growth economics mainly to two articles by James Tobin (1955, 1965). Tobin's 1955 article anticipated Solow and Swan by introducing factor substitution into Harrod-Domar type models and went farther by introducing monetary effects.

12. Stiglitz (1990), also a student of Solow, provides another account of growth economics at MIT in the 1960s.

13. Ramsey's insights were anticipated by Wicksell (see Boianovsky, this volume).

Despite incorporating a neoclassical production function, Tobin's main purpose, unlike Solow's or Swan's, was not to analyze steady-state growth but to investigate the interplay between money, nominal prices, growth, and fluctuations.

Tobin showed Solow a draft of his paper in November 1954. Solow replied in February 1955 and sent Tobin a first version of his own growth model:

> Way back in November I was absolutely fascinated when I read your paper on the train to Boston. When you look at the paper I am now sending you, you will see why. We had both been thinking almost exactly the same thoughts. . . . Strangely, there is practically no overlap between us, since you were interested mainly in the monetary and I in the real aspects. My paper at that time existed only in notes, and I decided it would hardly make any sense to send yours back until I could enclose a completed version. . . . Thank god I can *prove* I'm not a plagiarist—I gave a talk embodying essentially all of this paper at Chicago two weeks before I saw you at New Haven!

Solow's letter implies that he had set out the basic ideas of his growth paper around the end of October/beginning of November 1954 and wrote it in the first quarter of the next year. Tobin's first article on money and growth had only limited influence, probably, as Dimand and Durlauf observe, because it contained too much information. His 1965 paper received much more notice. With it, Tobin effectively initiated a new research program into the effects of monetary growth and inflation on the steady-state values of capital and output per worker. As an attempt to extend Keynesian economics into the long run, it was natural for Tobin to cite Harrod, Robinson, and Kaldor. While Tobin's model supports the neutrality of money, it also shows that a growing money supply can affect the real economy—that is, money is not *superneutral*. Sidrauski (1967) soon challenged Tobin's conclusion using an optimal monetary growth model.

Economic historians have studied the process of economic growth at least since the classic contributions of Alexander Gerschenkron, Simon Kuznets, and Walt Rostow in the 1950s. In his essay in this volume, Nicholas Crafts discusses the instrumental role of growth accounting methods—as developed and refined by Solow, Denison, and others—in the interpretation of key historical episodes (such as the Industrial Revolution) by modern quantitative economic historians. Economic historians have generally supported Solow's (1957) result that the growth of TFP is particularly important in explaining the development of old industrial economies, with the proviso that major technological changes affect the growth of output per worker only after an extended time lag. Things appear different for the relatively new industrialized countries, such as the so-called Asian tigers.

The growth accounting studies of Alwyn Young (1995) and others have provided evidence that factor accumulation—that is, increases in investment in capital and education, increases in labor force participation, and shift from agriculture to manufacturing—accounts for a large part of their growth. The evidence on newly industrialized economies, as Crafts observes, is consistent with Gerschenkron's hypothesis, which posits that the growth of relatively backward countries (as compared with the pioneers) depends mainly on capital accumulation, particularly when technology is able to flow relatively freely among countries. The heightened importance of factor accumulation relative to the "Solow residual" might be consistent with the neoclassical growth model as well, provided that historical experience can be interpreted as a dynamic transition path from one steady state to another with a higher rate of investment.[14]

Young's study in conjunction with versions of the neoclassical growth model that include human capital as a separate factor of production contributed to a "neoclassical revival" in growth theory in the 1990s. After Gregory Mankiw, David Romer, and David Weil's (1992) influential econometric study of the differences in growth rates among countries, cross-country econometric studies have often deployed an expanded neoclassical growth model to counter the claims of the so-called *new* growth theorists that only a model of endogenous technical progress could account for the historical record of national growth rates and income levels (Durlauf, this volume; see also Warsh 2006, 272–74, 319–21).

Gerschenkron (1952) first advanced the notion of "convergence" of incomes per capita among countries in elaborating his concept of "advantage of backwardness"—the hypothesis that relatively backward economies tend to grow at a faster rate than the first industrial countries because they borrow modern techniques of production and search for

14. The emphasis on transition dynamics became prominent in the early 1990s. While they were not stressed in the original formulations of the Solow and Swan models, some of the elements can be found already in Solow's 1969 Radcliffe Lectures.

"substitutes for prerequisites" for the productive factors, internal demand, or institutions that they lack. The neoclassical growth model also makes a prediction about convergence, albeit based on a different framework (diminishing returns to capital). Among countries that have the same steady state, those with current lower-income per capita should grow faster than rich countries and eventually catch up.

Convergence (or the lack of it) was not on Solow's or Swan's original agenda. Both restricted their investigation to the growth path within a single economy, without much concern about international comparisons. Indeed, while discussing Kaldor's stylized facts, Solow ([1970] 2000, 3) remarked that facts relating to differences between economies—such as the variety of growth rates across countries—were largely alien to his model. In their contributions to this volume, John Toye and Brian Snowdon note that the questions posed by pure growth theory in the 1950s and 1960s were generally distinct from the ones posed in development economics, which emerged as a field around the same time (e.g., Lewis 1954).

The neoclassical growth model was originally designed for a closed economy—under the double assumption of constant returns to scale and exogenous technological progress. Trade was not modeled as an important factor for economic growth. However, once increasing returns are introduced, the scale effects of international trade become crucial in explaining growth. Solow notes in his keynote address that, despite a few important contributions (especially Helpman and Krugman 1985, and Grossman and Helpman 1991), open economy growth theory has not attracted wide attention. Convergence of levels of income per head is explained with the standard neoclassical model by common technology, savings rate, and population growth. When a technological laggard gets access to more-advanced technology, a fast rate of growth is possible until the steady state is achieved and both countries share the same per capita income level and growth rate. Trade theory provides a second mechanism for convergence—factor price equalization as predicted by Eli Heckscher and Bertil Ohlin. Toye argues that trade theory and policy has thus significantly influenced (old and new) development economics.

More recently, (modified versions of) the neoclassical growth model have been used to study convergence and poverty traps, helping diminish the gap between growth and development economics. Solow (1956, 90) showed that, when there is some absolute minimum to consumption per capita and endogenous population growth, multiple equilibria are possible.

The idea of such multiple equilibria has played an important role in modern debates about poverty traps and foreign aid.[15] Snowdon also notes studies of the role of institutions in the growth process as another connection between growth theory and economic development (see also Solow 2005, 6).

The scarcity of natural resources is an important aspect of the growth process that affects developed and underdeveloped countries alike. Scarce natural resources, of course, were a crucial element of the classical Malthusian approach to growth in the early nineteenth century. Although Swan (1956) and, particularly, Meade (1961) had previously studied the analytic consequences of a finite supply of land for the neoclassical growth model, it was only in the 1970s that nonrenewable natural resources were integrated into growth economics (the topic is not mentioned in Hahn and Matthews's 1964 survey), even though the literature on the economics of exhaustible resources had existed at least since Harold Hotelling's (1931) contribution in the early 1930s. Guido Erreygers shows in his essay that it was the debate over the "limits to growth," together with the publication of John Rawls's *Theory of Justice*, that prompted Solow and others to develop the first growth models addressed to optimal capital accumulation with nonrenewable resources. In the early 1970s Solow criticized the Club of Rome report *Limits to Growth* for building models that, because they ignored substitution in the face of rising relative prices for scarce natural resources, could not help but bounce off the ceiling. In his keynote address to the *HOPE* conference, Solow suggests that the situation may be different nowadays because of large demands on natural resources associated with intensive growth in India, China, and other densely populated countries.

III

One of the main conclusions of the Solow-Swan model was that, under the assumption of diminishing returns to capital, the steady-state rate of growth of income per capita is governed by the rate of technological

15. Another source of multiple equilibria in Solow's 1956 model was the convexity of the production function provoked by increasing returns to capital at low levels of the capital stock. This would lead to a positive relation between the capital-output ratio (and so the rate of increase of capital) and the capital stock, similarly to the insights of early development economists.

progress.[16] Solow (see, e.g., [1970] 2000, 98) acknowledges that the theory left an obvious gap, since technological progress is an exogenous variable not explained within the model. What is more, if technology were regarded as an input in an economy that otherwise displays constant returns to scale in labor and capital, the economy would exhibit increasing returns in respect to all factors. As Kaldor (1961) first pointed out, in the face of such nonconvexities, not all factors can be paid their marginal products as is typically assumed in the analysis of perfect competition. In his classic paper on "learning by doing," Arrow (1962) suggested that the growth of technical knowledge was an unintended consequence of the experience of producing new capital goods and, therefore, external to the firms. Kaldor's (1957) "technical progress function" was an earlier attempt to capture a similar insight.[17] The alternative would be to deploy imperfectly competitive models, but that option was not open to growth economists at the time (Stiglitz 1990). Things started to change when Romer (1990) argued that ideas are nonrival, partially excludable goods, which implies the presence of increasing returns and the existence of monopolistically competitive rents used to pay for the resources used in the generation of innovations.[18]

Historically, the notion that economic growth is largely the result of increasing returns was first forcefully advanced in Allyn Young's 1928 address, which was a reelaboration of Adam Smith's theme about the

16. There may be another way to offset the diminishing returns to capital accumulation in the neoclassical growth model. If the aggregative elasticity of substitution between capital and labor is large enough (higher than one), the capital-labor ratio will increase indefinitely, accompanied by sustained growth in income per head (Solow 1956, 77; 2005, 8–9).

17. See Hahn and Matthews 1964, sec. 2.4, for a contemporary overview of the attempts of learning models to deal with growth and technical progress in a competitive framework. Solow, in his Arrow Lectures delivered at Stanford in 1993 and published in 1997, would later examine learning by doing in detail. In both Arrow's and Kaldor's models the steady-state rate of growth was, as in the Solow-Swan model, independent of savings behavior. They were not, therefore, "endogenous" in that sense. Erik Lundberg and some other economists trained in the Harrod-Domar tradition were dismayed by that result. In a contribution that went largely unnoticed until recently, Marvin Frankel (1962) advanced the first model of "endogenous" growth of the *AK* variety with constant returns to capital because of externalities at the firm level (Aghion and Howitt 1998, 26–27; Cannon 2000; see also Pomini and Tondini 2006).

18. The insight that nonrival knowledge is a main source of increasing returns may be found already in J. Maurice Clark's (1923, 119–23) treatment of overhead costs: "In a sense, knowledge is the only instrument of production that is not subject to diminishing returns. . . . The same research department can serve a large plant as well as a small one. Indeed, in technical matters where a law, once learned, is universal, one laboratory could serve the entire business of the country. . . . The costs of intellectual equipment, then, are one of the big sources of economy in large-scale production."

dynamic role of the division of labor. Young's approach to the growth process influenced the first generation of development economists in the 1940s and 1950s, but—with a few exceptions such as Svennilson and Kaldor—not growth theorists. Hahn and Matthews (1964, sec. 2.2) clarified the conditions under which increasing returns can be readily incorporated into a steady-state growth model. Output and capital will grow at the same rate, which is a multiple of the rate of growth of population. Even in the absence of technical progress, the economies of scale will bring about a permanent increase in income per capita so long as the rate of population growth is positive.[19]

Young's central message, on the other hand, was that progress is determined by the increased specialization of labor across a growing variety of goods—the extent to which capital is used in relation to labor is mainly governed by the scale of operations, that is, the capital-labor ratio depends on a larger extent on the size of the market rather than on factor prices. Young's notion of increasing returns through continuing specialization in production would have a marked influence on Kaldor's interpretation of growth as a disequilibrium process after the 1960s (Wulwick 1993) and on Romer's (1990) modeling of technological progress as creating new varieties of capital goods. However, as Roger Sandilands (this volume) argues, Romer's formalization leaves out of the picture important features of Young's notion of macroeconomic increasing returns and the role of the growth of demand in deciding the scope for application of knowledge to the productive process.[20]

Whereas Young stressed increasing returns, Joseph Schumpeter—another crucial intellectual source of non-steady-state growth economics—argued that growth results from technological progress through "creative destruction." Schumpeter taught Solow at Harvard in the 1940s, but made no lasting impression on him. In particular, although Solow's (1956) model may look like a vindication of Schumpeter's insight about the pivotal role of technical change in the growth process, there is no real connection between their analyses.

19. Such "scale effects" are a controversial feature of several growth models, e.g., Arrow 1962 and Romer 1990. Solow ([1970] 2000, 113) points out that just allowing for increasing returns is not enough to generate a fully "endogenous" growth model—in the sense that the rate of growth is explained within the model—since the long-run rate of growth is still exogenous.

20. While commenting at the *HOPE* conference on Young's notion that growth generates growth in cumulative fashion, Solow pondered whether there is no analogue to diminishing returns in specialization.

Schumpeter's approach, as William Baumol (this volume) reminds us, was especially relevant for the microeconomics of technological progress, based on the study of competition between firms through innovation. Although Schumpeter's approach influenced important aspects of new growth theory (see especially Aghion and Howitt 1998), a full formal investigation of entrepreneurship is still missing. From a methodological standpoint, one should distinguish between macroeconomic growth theory—the study of the long-run performance of the economy conditioned on the evolution of technology—and the microeconomic investigation of the process of technological change (Solow [1970] 2000, 101).

Solow's main criticism of endogenous new growth theory—partly supported by Baumol—is the special character of the assumption that the production function for ideas is a linear differential equation. The "linearity critique" (Jones 1998, chaps. 5, 8) has played an important role because, among other reasons, linearity ensures that, in contrast to the conclusions of the Solow-Swan model, changes in economic policy that affect, say, the savings rate can permanently increase the growth rate of output.

Twenty years after their first appearance, none of the new growth theory models or any synthesis model has definitively supplanted the old neoclassical growth model. Durlauf (this volume) ascribes the failure to the "open-endedness" of growth models—that is, to the mutual compatibility of different growth theories.[21] Another factor is undoubtedly the difficulties in adequately testing the models against data. Although the recent literature on economic growth is more empirically orientated than the older literature—as witnessed especially by the boom in cross-country growth regressions after the early 1990s—the methodological role of growth econometrics is still an open issue. Durlauf suggests that the relationship between new growth theory and growth empirics will tend to mirror that between neoclassical growth theory and data, with growth regressions taking over the role of growth accounting as a supplier of stylized facts that models need to capture.

The interpretation of one particular piece of empirical evidence—the Solow residual—has played an important role in the controversy about the interconnection between short- and long-run macroeconomics. Beginning in the 1980s, real business cycle theorists have argued that, at business

21. Solow (2005, 4) has claimed that "there is not really any competing model" to the basic neoclassical growth model, since, in a broad sense, the new growth theories are completely neoclassical. See Helpman 2005 for a different opinion.

cycle frequencies, total factor productivity is strongly correlated with output and hours worked. Procyclical labor supply and investment propagate shocks to total factor productivity (TFP), generating fluctuations in output and labor productivity in economy well described by an optimal growth model. New Keynesian macroeconomists have challenged this interpretation of the residual. They argue that short-run fluctuations of TFP reflect departures from perfect competition and from constant returns to scale.

In the final essay in this volume, Tiago Mata and Francisco Louçã recount the debate. They argue that, as often happens in economics, the concept of the residual has been used in ways quite distinct from the original context in which it was formulated. While Solow himself has not been deeply engaged in this controversy, he did recall in his reaction to the contributions to a Festschrift in his honor that in his 1957 article he was interested in the trend, not in year-to-year oscillations, of TFP, but, if asked about the meaning of such yearly fluctuations, he would largely side with the new Keynesians in emphasizing increasing returns in the short run, when output is below capacity (Solow 1990, 224–26).

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Part 1
Growth Economics in the First Person