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Relating Period and Cohort Fertility

Robert Schoen

ABSTRACT From a population perspective, the trajectories of both the total fertility at successive time periods and the total fertility of successive birth cohorts are derived from the same array of age-specific fertility rates. This analysis uses the assumption of constant age-specific fertility proportions to derive new explicit relationships between period and cohort fertility. In short, period total fertility is approximately equal to the total fertility of the cohort born a generation earlier, with a modest additive adjustment. A simple relationship also links both period and cohort total fertility to ACF, the average fertility of the childbearing cohorts in a given year. Assuming that fertility levels follow a cubic curve, cohort values from the derived relationships are then compared to observed cohort fertility values for the United States in 1917–2019. Despite substantial violations of the constant proportional fertility assumption, the calculated values deviate from the observed values by an average of only 7–8%. Short-term projections suggest that U.S. cohort fertility will continue to decline.

KEYWORDS Period fertility • Cohort fertility • Fertility translation • Mean age of fertility • Average cohort fertility

Introduction

Ryder's (1964) defining article on "translation" in the first issue of *Demography* cast period fertility fluctuations as distortions of the underlying cohort behavior. Ryder (1964, 1965) saw the cohort, that is, the group of persons born in the same period, as a key sociological concept because social change has different impacts on persons of different ages, and the consequences of those impacts persist. In the study of fertility, Ryder believed the cohort was theoretically central because the behavior of different cohorts reflects those different socioeconomic conditions, and empirically significant because cohort fertility is more stable than period fertility. That translation perspective has since been updated by Keilman (2001) and further elaborated in Zeng and Land (2002).

In contrast to that cohort-centric view, Ní Bhrolcháin (1992) offered a period-centered perspective. She argued that period changes were unambiguously the prime source of fertility variation, and "if there are cohort effects in twentieth-century developed-country fertility series, they are so subtle as to be extremely difficult to detect" (Ní Bhrolcháin 1992:662). The empirical review by Hobcraft et al. (1982)

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saw consistency in the age pattern of *period* fertility but not in the age pattern of cohort fertility, indicating a greater consistency across age in period behavior. That suggests that the greater stability in cohort fertility is simply structural, in that cohort fertility just averages the highs and lows of period experience.

Cohort fertility has a clear heuristic appeal: women have children, mostly one at a time, and their past childbearing is obviously relevant to their subsequent behavior. Still, people live day by day, and those period influences can be long-lasting. The cohort emphasis ignores the fact that period fertility determines cohort size. There is also the practical limitation that the cohorts of greatest interest—those in their peak reproductive years—are decades away from completing their childbearing, and there is no generally accepted way of completing their fertility (Bohk-Ewald et al. 2018). Two decades ago, the issue of period fertility "distortions" arose again, with the controversy over whether recently observed period fertility declines actually reflected a change in cohort fertility behavior (Bongaarts and Feeney 1998; Frejka and Calot 2001; Kim and Schoen 2000; Lesthaeghe and Willems 1999; Schoen 2004; Van Imhoff and Keilman 2000).

Translation from period to cohort fertility can potentially help resolve such methodological questions. Translation adjustments to period fertility measures were first proposed by Ryder (1964), and a further noteworthy adjustment was proposed by Bongaarts and Feeney (1998), although both of those procedures use strong assumptions to relate cohort and period fertility.

To be specific, Ryder (1964) explored expansions of fertility functions in terms of moments in order to translate between period and cohort measures. His main result was the linear (first moment) relationship

$$\mathrm{TFR}(T+A_c(T)) = \mathrm{CFR}(T)\,[1-\Delta A_c(T)], \tag{1}$$

where TFR indicates a period total fertility rate, CFR a cohort total fertility rate, Δ a change or first derivative over time, A_c a cohort mean age of fertility, and T the birth year of a cohort. A total fertility rate is the sum of the age-specific fertility rates of a period (or cohort). For clarity, time index T denotes a cohort year of birth while t denotes period (or year) t. Thus $\mathrm{TFR}(T+A_c(T))$ represents the period TFR in the year that the cohort born in year T is at its mean age of fertility.

Bongaarts and Feeney (1998) adopted a period perspective and proposed the alternative linear relationship

$$\mathrm{TFR}^*(t) = \mathrm{TFR}(t)\,/\,[1-\Delta A_p(t)], \tag{2}$$

where $\mathrm{TFR}^*(t)$ is the Bongaarts–Feeney adjusted TFR designed to eliminate period distortions, and A_p is the period mean age of fertility. Bongaarts and Feeney (1998) made their adjustment parity specific, but Eq. (2) captures the essence of their approach. They asserted that their TFR* better reflected the completed family size (i.e., cohort fertility) than did the observed period total fertility rates. Zeng and Land (2002:270) clearly expressed the assumptions underlying the Bongaarts and Feeney TFR*, that is, TFR* was the CFR of the hypothetical cohort whose fixed rate schedule shifted each reproductive year by a fixed amount. Thus TFR* rests on both a fixed fertility rate schedule and 30–35 years of constant upward (or downward) shifts in that schedule.

Such linear-based adjustments are at best approximations to the actual period/cohort relationship. The goal of this article is to further explore and quantify period/cohort relationships and, from that perspective, examine recent trends in U.S. period and cohort fertility.

Period/Cohort Relationships Under Constant Proportional Fertility

The Constant Proportional Fertility Model

To begin, assume a constant proportional age schedule of fertility rates in every year, while the level of fertility is free to vary from period to period. In the context of such a model, we can relate period and cohort fertility based on the moments of the constant age-specific fertility proportions and a known function that specifies the fertility level over time. By doing so, both period and cohort fertility can be expressed in terms of a few underlying parameters.

We build a model where the relationships between period and cohort fertility are stated explicitly. At time t, let the fertility rate at exact age x be $f(x,t)$. Assume no mortality before age 45, and let the reproductive ages be from 15 through 44. Denote the fixed proportion of period fertility at age x by $c(x)$, with $\int c(x)\,dx = 1$. Unless otherwise specified, integrals range from age 15 to age 45. At time t, let those fixed fertility proportions be multiplied by fertility level function TFR(t), so time t fertility at age x is given by $f(x,t) = \text{TFR}(t)\,c(x)$, with the sum of the time t age-specific fertility rates equal to TFR(t).

The constant proportional fertility assumption is somewhat strong, and the period mean age of fertility (A_p) has risen considerably in many places over the past several decades. Nonetheless, the Hobcraft et al. (1982) finding that the age pattern of fertility is quite stable provides a reasonable point of departure. In the absence of plausible constraints, period/cohort relationships would be virtually impossible to analyze. The stable population, the dominant model of formal demography, is based on constant age-specific fertility and mortality rates. Despite those very strong and unrealistic assumptions, the stable model has been very useful in analytical work and in estimating demographic measures from incomplete data (*cf.* United Nations 1983). Unlike the Ryder and Bongaarts–Feeney analyses, constant fertility proportions allow fertility levels to vary freely over time.

The United Nations 2014 population projections used a modified constant fertility proportions approach, with a population's initial age-specific proportions moving to fixed ultimate proportions (Ševčíková et al. 2016:301–302). The recent comprehensive analyses of fertility projection and cohort completion by Bohk-Ewald et al. (2018) found that more complex projection methods do not necessarily do better than simpler methods. In particular, they found that the assumption of constant ("frozen") rates "consistently outperforms most of the sophisticated and less sophisticated forecast methods" (Bohk-Ewald et al. 2018:9191). With TFR(t) able to vary freely and the $c(x)$ specific to the case at hand, the constant fertility proportions assumption is flexible, demographically grounded, and far weaker than assumptions frequently employed in empirical demographic modeling. Furthermore, the primary objective here is to relate period and cohort fertility, not to improve techniques for population projection. Assuming constant fertility proportions allows new, explicit relationships to be derived that directly link the total fertility of a year to the completed fertility of a cohort. The best-performing methods in Bohk-Ewald et al. (2018), which use time series/extrapolations or Bayesian methods, do not provide such direct period/cohort relationships.

Now let us relate period and cohort fertility when TFR(t) is a known polynomial function. With CFR(T) the total fertility (or completed family size) of the cohort born at time T, we can write

$$\begin{aligned} \mathrm{CFR}(T) &= \int \mathrm{TFR}(T+x)\, c(x)\, dx \\ &= \int f(x, T+x)\, dx, \end{aligned} \tag{3}$$

where the integral ranges over the 30-year reproductive age span. Equation (3), our starting point, provides the basic relationship between period and cohort fertility.

Relationships Under a Linear Fertility Trajectory

Explicit relationships can now be derived for specific fertility trajectories. When the fertility trajectory is linear, we can write

$$\mathrm{TFR}(t) = R + at, \tag{4}$$

where parameter R is the level of fertility at time 0 and parameter a is the linear slope. Then, from Eq. (3), with integration from ages 15 to 45, we can write

$$\begin{aligned} \mathrm{CFR}(T) &= \int \mathrm{TFR}(T+x)\, c(x)\, dx = \int [R + a(T+x)]\, c(x)\, dx \\ &= R + aT + a \int x\, c(x)\, dx = R + a(T+\mu) = \mathrm{TFR}(T+\mu), \end{aligned} \tag{5}$$

where μ, the mean of the $c(x)$ distribution, equals $\int x\, c(x)\, dx$. With a linear time trajectory, the completed family size of the cohort born in year T is *identical* to TFR($T+\mu$), the total fertility rate of the year ($T+\mu$).

Figure 1 shows period and cohort age curves of fertility when $R=1$ and $a=0.02$. To simplify the calculations, the constant proportional age schedule of fertility is given by the parabola

$$c(x) = (-3 / 20) + x / 75 - x^2 / 4500 \tag{6}$$

between the ages of 15 and 45 and is zero at all other ages. The derivation of the curve is straightforward and described in section D of the online appendix and Appendix Supplement 1. The $c(x)$ curve is zero at ages 15 and 45 and has an area of 1 between those ages. It is symmetric, with a mean at age 30, where $c(30) = 0.05$, a variance of 45, and zero skew. While it simplifies the typical age curve of fertility, that parabolic curve affords a reasonable depiction of actual behavior.

In Figure 1, both period ($t=30$) and cohort ($T=0$) total fertility are 1.6. The two curves cross over at exact age 30. Below age 30, period fertility is higher; after age 30, cohort fertility is higher. More specifically, the excess of period fertility over cohort fertility at age $30-k$ ($0 < k < 15$) exactly equals the excess of cohort fertility over period fertility at age $30+k$. Section D in the online appendix gives the values underlying Figure 1 for five-year age-groups, showing that symmetry.

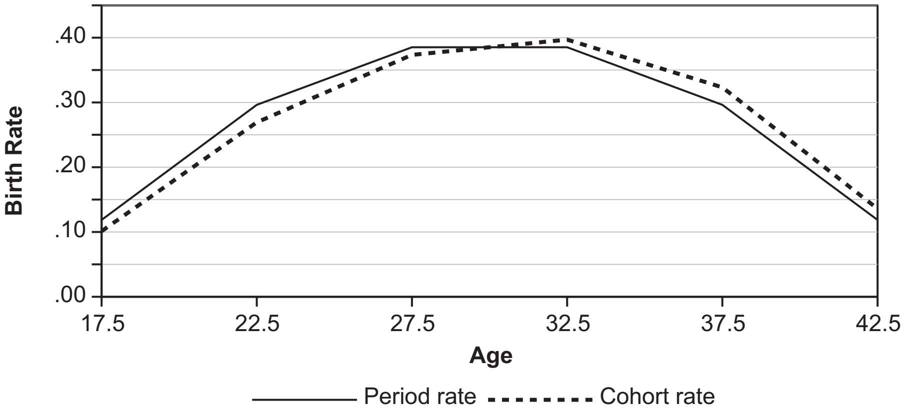

Fig. 1 Period and cohort age-specific birth rates with parabolic period rates and linear fertility change over time

The mean ages of fertility are also significant functions, and their values can readily be found. With constant $c(x)$ and a linear change in fertility, the period mean age of fertility (A_p) is always μ, while the cohort mean age, A_c, is given by

$$A_c(T) = \mu + a \text{ Var} / \text{CFR}(T) = \mu + a \text{ Var} / \text{TFR}(T+\mu), \tag{7}$$

where $A_c(T) = \int x\, c(x)\, [R + a(t+x)]\, dx$ and Var denotes the variance of the $c(x)$ distribution, that is, $\int (x-\mu)^2\, c(x)\, dx$. The full derivation of Eq. (7) is given in section A of the online appendix. Unlike the period mean age, the cohort mean age varies over time, differing from μ by a combination of the slope of the linear trajectory (a), the dispersion of the age pattern of fertility (Var), and the total fertility of the cohort.

The Ryder (1964) linear translation relationship in Eq. (1) is consistent with Eq. (5), when the *cohort* mean age of fertility is used. Employing Eqs. (1), (4), (5), and (7), the *left* side of Eq. (1) can be written

$$\begin{aligned} \text{TFR}(T + A_c(T)) &= R + a[T + A_c(T)] = R + a\{T + \mu + a \text{ Var} / [R + a(T+\mu)]\} \\ &= [R + a(T+\mu)] + a^2 \text{ Var} / [R + a(T+\mu)] \\ &= \text{TFR}(T+\mu) + [a^2 \text{ Var} / \text{TFR}(T+\mu)]. \end{aligned} \tag{8}$$

To see that the right side of Eq. (1) also yields the result in Eq. (8), first find the change in $A_c(T)$, that is,

$$dA_c(T) / dT = -a^2 \text{ Var} / [R + a(T+\mu)]^2. \tag{9}$$

Using Eq. (5) and substituting the relationship in Eq. (9) in the *right* side of Eq. (1) yields

$$\begin{aligned} \text{TFR}(T + A_c(T)) &= [R + a(T+\mu)]\{1 + a^2 \text{ Var} / [R + a(T+\mu)]^2\} \\ &= [R + a(T+\mu)] + a^2 \text{ Var} / [R + a(T+\mu)] \\ &= \text{TFR}(T+\mu) + [a^2 \text{ Var} / \text{TFR}(T+\mu)], \end{aligned} \tag{10}$$

where the last equality in Eq. (10) equals that in Eq. (8).

It is easier to show that the Bongaarts–Feeney Eq. (2) is consistent with Eq. (5) for the *period* mean age of fertility**.** There is no adjustment, as μ is constant. Hence, at time $t = T + \mu$, we have $\text{TFR}^*(t) = \text{TFR}(t) = \text{TFR}(T+\mu) = \text{CFR}(T)$.

The consistency of results in the linear case between the Ryder and the Bongaarts–Feeney approaches on one hand and the constant $c(x)$ approach on the other hand reinforces the plausibility of the constant proportional fertility assumption. Since the constant $c(x)$ approach can be applied to nonlinear fertility trajectories, it extends those earlier approximations and opens the door to more general relationships.

Relationships Under a Quadratic Fertility Trajectory

When period fertility follows a quadratic curve, we can write

$$\mathrm{TFR}(t) = R + at + bt^2. \tag{11}$$

Then, integrating using Eqs. (3) and (11) gives

$$\mathrm{CFR}(T) = \mathrm{TFR}(T + \mu) + b \text{ Var} \tag{12}$$

and

$$A_c\,(T) = \mu + \{\text{Var }[a + 2b(T + \mu)]\}\ /\ \mathrm{CFR}(T). \tag{13}$$

Section B of the online appendix shows the details of the derivation of Eqs. (12) and (13).

In the quadratic case, the difference between CFR(T) and TFR($T+\mu$) is a constant, the extent of nonlinearity in the TFR trajectory times the variance of the $c(x)$ distribution. The difference between the period and cohort mean ages varies nonlinearly over time.

Relationships Under a Cubic Fertility Trajectory

When fertility levels follow a cubic curve, let us write

$$\mathrm{TFR}(t) = R + at + bt^2 + dt^3. \tag{14}$$

Proceeding as before, section C of the online appendix shows that we then have

$$\mathrm{CFR}(T) = \mathrm{TFR}(T + \mu) + \text{Var }[b + 3d(T + \mu)] \tag{15}$$

and

$$A_c(T) = \mu + \text{Var }\{a + 2b(T + \mu) + 3d\,[(T + \mu)^2 + \text{Var}]\}\ /\ \mathrm{CFR}(T). \tag{16}$$

When the level of fertility follows a cubic curve, the difference between CFR(T) and TFR($T+\mu$) varies linearly over time. If parameter d equals zero, that is, if the fertility trajectory is quadratic, Eqs. (15) and (16) reduce to Eqs. (12) and (13). A cubic curve is generally considered a reasonable approximation for most observed trajectories over the short to medium range (e.g., Hoem and Linnemann 1988). Miller's classic actuarial text on graduation goes so far as to state that "over a limited range, most regular series met with by the actuary may be closely approximated by a polynomial of the third degree" (Miller 1946:26).

Table 1 Period/cohort relationships with constant age-specific fertility proportions and polynomial time trajectories of fertility levels

Measure	Period(t)	Cohort(T)
A. Fertility Level Varies Linearly Over Time		
TFR	$R+at$	$R+a(T+\mu)=\text{TFR}(T+\mu)=\text{CFR}(T)$
A	μ	$\mu+a$ Var / CFR(T)
B. Fertility Level Varies Quadratically Over Time		
TFR	$R+at+bt^2$	TFR($T+\mu$)$+b$ Var
A	μ	$\mu+[a+2b(T+\mu)]$ Var / CFR(T)
C. Fertility Level Varies Cubically Over Time		
TFR	$R+at+bt^2+dt^3$	TFR($T+\mu$)$+[b+3d(T+\mu)]$ Var
A	μ	$\mu+\{a+2b(T+\mu)+3d[(T+\mu)^2+\text{Var}]\}$ Var / CFR(T)

Notes: The time 0 fertility level (TFR(0)) is set at R. With $c(x)$ being the constant fertility proportion at age x, μ is the mean of the $c(x)$ distribution and Var is its variance. In period measures, t indicates a current year. In cohort measures, time T is the cohort's year of birth. CFR denotes a cohort total fertility rate, and A denotes a mean age of fertility.

The quadratic and cubic derivations provide new period/cohort relationships that go beyond the linear relationships in Ryder (1964) and Bongaarts and Feeney (1998). Those new relationships indicate that cohort fertility levels and mean ages are the period values for the time when the cohort is at the period mean age, adjusted *additively* for nonlinearity in the trajectory of period fertility and for the mean and variance of the fixed age pattern of fertility. Table 1 summarizes those period/cohort relationships for linear, quadratic, and cubic fertility trajectories.

Relationships in Hypothetical Data With a Cubic Trajectory

Calculations with hypothetical data can illustrate the magnitude of those additive adjustments. Table 2 shows fertility levels and mean ages when fertility follows a cubic trajectory with either low or high fluctuations. The low-fluctuation pattern, roughly based on twentieth-century Swedish experience, has TFR varying from a high of 2.31 to a low of 1.54. The high-fluctuation pattern, somewhat similar to the United States in the twentieth century, has TFRs varying from 3.0 to 1.8.

With low fluctuations, the difference between CFR(T) and TFR($T+\mu$) is rather small, on the order of one hundredth of a child. That difference is precisely the adjustment to TFR($T+\mu$) in Eq. (15). Figure 2 shows period and cohort total fertility trajectories under both scenarios. The period and cohort curves are distinct, but quite similar. With high fluctuations, period/cohort differences are larger, but completed family sizes differ by only ±0.1 child.

The difference between cohort mean age A_c (T) and period mean age μ is more substantial. With low fluctuations, it goes from nearly –0.5 to over +0.3, and with high fluctuations from –1.7 to +0.2. That difference is precisely the additive adjustment to μ in Eq. (16). In short, a linear trend assumption fails to capture the complexities of the cubic case, although there is still a considerable closeness between the values for the cohort born in year T and the period $T+\mu$.

Table 2 Period and cohort fertility measures from hypothetical data with constant proportional age-specific fertility and fertility levels following a cubic trajectory

	A. Low-Fertility Fluctuations					B. High-Fertility Fluctuations				
Time (t)	TFR(t)	CFR(T)	[CFR(T)–TFR(t)]	$A_c(T)$	A_c-30	TFR(t)	CFR(T)	[CFR(T)–TFR(t)]	$A_c(T)$	A_c-30
15	2.310	2.317	0.007	29.55	–0.45	3.000	3.080	0.080	28.98	–1.02
20	2.198	2.205	0.007	29.56	–0.44	2.701	2.772	0.071	29.14	–0.86
25	2.093	2.101	0.007	29.57	–0.43	2.481	2.544	0.063	29.32	–0.68
30	1.997	2.005	0.008	29.59	–0.41	2.331	2.385	0.054	29.52	–0.48
35	1.909	1.917	0.008	29.61	–0.39	2.241	2.286	0.045	29.71	–0.29
40	1.830	1.838	0.008	29.63	–0.36	2.200	2.236	0.036	29.89	–0.11
45	1.760	1.768	0.008	29.67	–0.33	2.200	2.228	0.028	30.03	0.03
50	1.699	1.708	0.008	29.70	–0.30	2.230	2.249	0.019	30.13	0.13
55	1.648	1.656	0.009	29.75	–0.25	2.281	2.291	0.010	30.20	0.20
60	1.606	1.615	0.009	29.79	–0.21	2.344	2.345	0.001	30.21	0.21
65	1.574	1.583	0.009	29.85	–0.15	2.407	2.400	–0.007	30.20	0.20
70	1.552	1.561	0.009	29.90	–0.10	2.463	2.446	–0.016	30.14	0.14
75	1.540	1.550	0.010	29.96	–0.04	2.500	2.475	–0.025	30.06	0.06
80	1.539	1.549	0.010	30.03	0.03	2.509	2.476	–0.034	29.94	–0.06
85	1.549	1.559	0.010	30.09	0.09	2.481	2.439	–0.042	29.78	–0.22
90	1.569	1.579	0.010	30.15	0.15	2.406	2.355	–0.051	29.58	–0.42
95	1.601	1.612	0.010	30.21	0.21	2.274	2.214	–0.060	29.30	–0.70
100	1.645	1.655	0.011	30.27	0.27	2.075	2.006	–0.069	28.90	–1.10
105	1.700	1.711	0.011	30.32	0.32	1.800	1.722	–0.078	28.30	–1.70

Notes: Fertility trajectory from Eq. (14), with mean (μ)=30 and variance (Var)=45. Time $T=t-30$. Cubic curves were fitted to TFR(15)=2.31, TFR(45)=1.76, TFR(75)=1.54, and TFR(105)=1.70 for the low-fluctuation pattern (with b=0.00014167, d=0.00000030864) and to points TFR(15)=3.0, TFR(45)=2.2, TFR(75)=2.5, and TFR(105)=1.80 for the high-fluctuation pattern (with b=0.00236111, d=–0.00001296). See online Appendix Supplement 2 for further details.

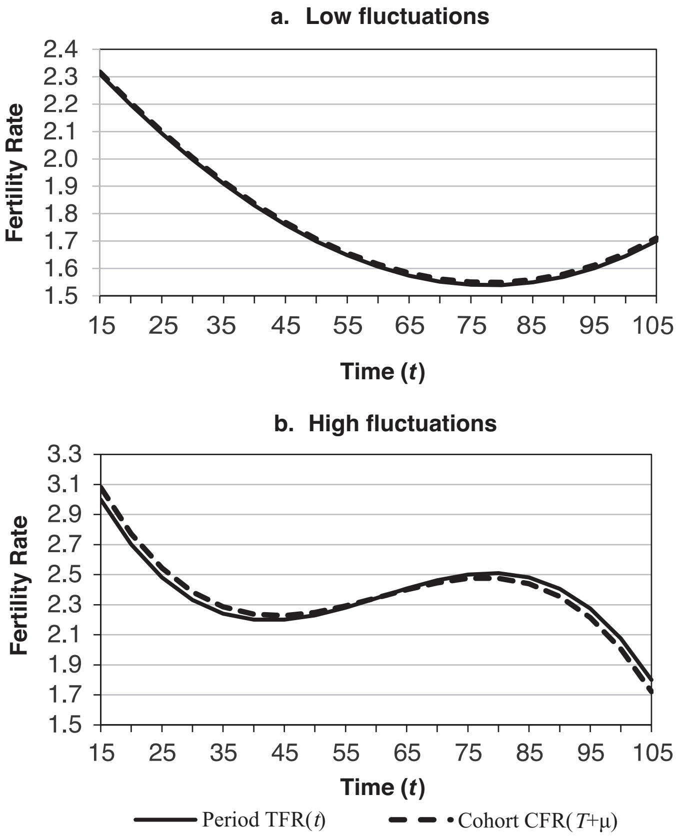

Fig. 2 Period and cohort total fertility under cubic fertility trajectories

Average Cohort Fertility With Constant Proportional Fertility

Butz and Ward (1979), in examining the effects of timing on period and cohort fertility, introduced the average cohort fertility (ACF) concept. As a weighted average of the fertility of cohorts childbearing at time *t*, the ACF provides a period measure that offers insight into cohort behavior. Under the constant proportional fertility assumption, let us write

$$\begin{aligned} \mathrm{ACF}(t) &= \int \mathrm{CFR}(T+\mu-x)\,[\mathrm{TFR}(t)\,c(x)]\,dx \,/\, \mathrm{TFR}(t) \\ &= \int \mathrm{CFR}(T+\mu-x)\,c(x)\,dx \end{aligned} \tag{17}$$

with $t=T+\mu$ and the integral spanning the ages 15 to 45. The definition in Eq. (17) differs from that of Butz and Ward (1979) in that the weights are the fraction of period (not cohort) fertility at each age.

Now assume that period total fertility varies cubically over time per Eq. (14), so cohort total fertility varies according to Eq. (15). Our period-weighted ACF can then be written

$$\mathrm{ACF}(t) = \int [\mathrm{TFR}(t+\mu-x) + \mathrm{Var}\,(b+3d\{t+\mu-x\})]\,c(x)\,dx. \tag{18}$$

As shown in section E of the online appendix, straightforward integration over the range of $c(x)$ yields

$$\mathrm{ACF}(t)=\mathrm{TFR}(t)+2\ \mathrm{Var}\,(b+3dt)=\mathrm{CFR}(T)+\mathrm{Var}\,(b+3dt). \tag{19}$$

Equation (19) shows that under a regime of constant proportional fertility whose level varies cubically over time, there is a simple relationship between period-weighted ACF at any time and both period and cohort total fertility. From Eqs. (15) and (19), CFR(T) is the arithmetic mean of TFR(t) and ACF(t), that is,

$$\mathrm{CFR}(T)=[\mathrm{TFR}(t)+\mathrm{ACF}(t)]\,/\,2. \tag{20}$$

Equations (19) and (20) also hold when fertility varies quadratically, and if fertility changes linearly, that is, when $b=d=0$, ACF(t)=TFR(t)=CFR(T)=CFR($t-\mu$). These ACF relationships are new and afford another approach to period/cohort translation and to inferring cohort behavior from period measures.

Period/Cohort Relationships in American Fertility Data

Detailed fertility data for the United States go back to 1917, and those data provide an excellent source for examining the extent to which our model-based period/cohort relationships hold in an actual population over a long span of time. The data sources used are described in section F of the online appendix.

Table 3 shows the period TFR(t), the period A_p (t), and the CFR(T) for the years 1917 through 2019. Since the fertility data are by calendar year and the A_p values are not integral, a simplifying procedure was used to determine the appropriate birth cohort year $T=t-A_p$. Specifically, observed A_p fractions of ±(1/3) were ignored. If the fractional part of $A_p(t)$ was between 1/3 and 2/3, the fraction was assumed to be one half, and the CFR values for two years were averaged. For example, for an A_p (t) value of integral Z plus one half, CFR(T) was taken as ½[CFR($t-Z$)+CFR($t-Z-1$)].

In the continuous time models of Table 1, those born at exact time T are at exact age x at exact time $T+x$. With discrete data, those born in year T all attain exact age x in year $T+x$, although at age x last birthday they live some person-years during year $T+x+1$. To simplify matters, recognizing that single years of age and time are considered and that fractional mean ages of fertility are involved, the calculations assume that the behavior at age x of the cohort born in year T is captured by data for the year $T+x$.

Figure 3 shows the trajectories of TFR(t), CFR($t-A_p(t)$), and the cubic estimated CFR for the cohort attaining age A_p (t) in year t. The maximum U.S. period TFR was 3.68 in 1957. Fertility then fell to 1.74 in 1976 before rising again to 2.12 in 2007. In the following 12 years, fertility fell again, reaching an all-time U.S. low of 1.705 in 2019. Cohort fertility peaked at 3.18 for the cohorts born in 1933–1934, before falling to 1.98 for the cohort born in 1954. Subsequently, the CFR rose, reaching 2.24 for women born in 1978. For the most recent seven years, the CFR declined, but was still 2.08 for the cohort of 1984.

Since 1965, or the cohorts of 1938–1939, CFR(T) has generally been greater than TFR(t). That reflects the rise in the period mean age of fertility. From 25.74 in 1974, A_p (t) rose nearly monotonically to 29.62 in 2019, a rise of 3.88 years. Such a

Table 3 Summary measures of U.S. fertility, 1917–2019

Year (t)	TFR (t)	$A_p(t)$	CFR(T)	Year (t)	TFR (t)	$A_p(t)$	CFR(T)
1917	3.333	28.61	3.125	1965	2.882	26.55	2.962
1918	3.312	28.65	3.090	1966	2.670	26.41	2.885
1919	3.068	28.90	3.072	1967	2.526	26.32	2.745
1920	3.263	28.54	3.008	1968	2.431	26.22	2.635
1921	3.326	28.46	2.960	1969	2.423	26.15	2.520
1922	3.109	28.47	2.909	1970	2.432	26.04	2.422
1923	3.101	28.43	2.856	1971	2.245	25.98	2.332
1924	3.121	28.34	2.802	1972	1.994	25.89	2.264
1925	3.012	28.31	2.718	1973	1.862	25.80	2.196
1926	2.901	28.25	2.672	1974	1.835	25.74	2.138
1927	2.824	28.22	2.635	1975	1.744	25.75	2.084
1928	2.660	28.16	2.583	1976	1.738	25.83	2.048
1929	2.532	28.09	2.525				
1930	2.532	28.03	2.477	1977	1.790	25.86	2.017
1931	2.402	28.02	2.442	1978	1.760	25.91	1.994
1932	2.319	28.01	2.405	1979	1.808	25.94	1.980
1933	2.172	28.01	2.358	1980	1.840	25.94	1.978
1934	2.232	27.89	2.318	1981	1.812	26.04	1.982
1935	2.189	27.75	2.294	1982	1.828	26.10	1.986
1936	2.146	27.64	2.272	1983	1.799	26.18	1.988
				1984	1.806	26.27	1.993
1937	2.173	27.48	2.274	1985	1.844	26.32	2.003
1938	2.222	27.40	2.245	1986	1.838	26.37	2.009
1939	2.172	27.38	2.304	1987	1.872	26.46	2.020
1940	2.229	27.30	2.343	1988	1.934	26.49	2.027
1941	2.332	27.16	2.388	1989	2.014	26.47	2.036
1942	2.555	27.05	2.434	1990	2.081	26.52	2.048
1943	2.640	27.25	2.467	1991	2.062	26.53	2.060
1944	2.494	27.52	2.490	1992	2.046	26.53	2.078
1945	2.422	27.79	2.512	1993	2.020	26.59	2.100
1946	2.858	27.35	2.594	1994	2.002	26.69	2.111
1947	3.181	26.89	2.702	1995	1.978	26.80	2.125
1948	3.026	26.74	2.765	1996	1.976	26.89	2.127
1949	3.036	26.70	2.794				
1950	3.028	26.72	2.847	1997	1.971	26.99	2.131
1951	3.199	26.62	2.939	1998	1.999	27.15	2.168
1952	3.286	26.68	2.903	1999	2.008	27.25	2.207
1953	3.349	26.63	2.970	2000	2.056	27.39	2.216
1954	3.461	26.60	3.006	2001	2.030	27.52	2.229
1955	3.498	26.56	3.041	2002	2.020	27.71	2.233
1956	3.605	26.47	3.083	2003	2.048	27.87	2.218
				2004	2.052	27.95	2.240
1957	3.682	26.44	3.124	2005	2.057	28.00	2.240
1958	3.629	26.42	3.156	2006	2.108	27.97	2.243
1959	3.638	26.42	3.175	2007	2.120	27.99	2.221
1960	3.606	26.44	3.184	2008	2.072	28.06	2.183
1961	3.564	26.48	3.181	2009	2.002	28.17	2.154
1962	3.423	26.47	3.155	2010	1.931	28.35	2.150
1963	3.298	26.49	3.104	2011	1.894	28.54	2.134
1964	3.171	26.55	3.036	2012	1.880	28.67	2.122

Table 3 (continued)

Year (t)	TFR (t)	$A_p(t)$	CFR(T)	Year (t)	TFR (t)	$A_p(t)$	CFR(T)
2013	1.858	28.84	2.081	2017	1.766	29.40	—
2014	1.862	29.01	—	2018	1.730	29.56	—
2015	1.844	29.15	—	2019	1.705	29.62	—
2016	1.820	29.32	—				

Notes: See text for further details. In determining CFR(T), fractional A_p values of ±(1/3) were ignored. Fractional values between 1/3 and 2/3 were set to ½, and CFR values for two years were averaged. Cohort experiences at ages over 35 in 2019 were completed using published age-specific fertility rates for 2019.

Sources: Period and cohort fertility values were taken or calculated from Heuser (1976), Hamilton and Cosgrove (2010), Hamilton and Kirmeyer (2017), Martin et al. (2017), Martin et al. (2019), and Hamilton et al. (2020). See Sections F and G in the online appendix.

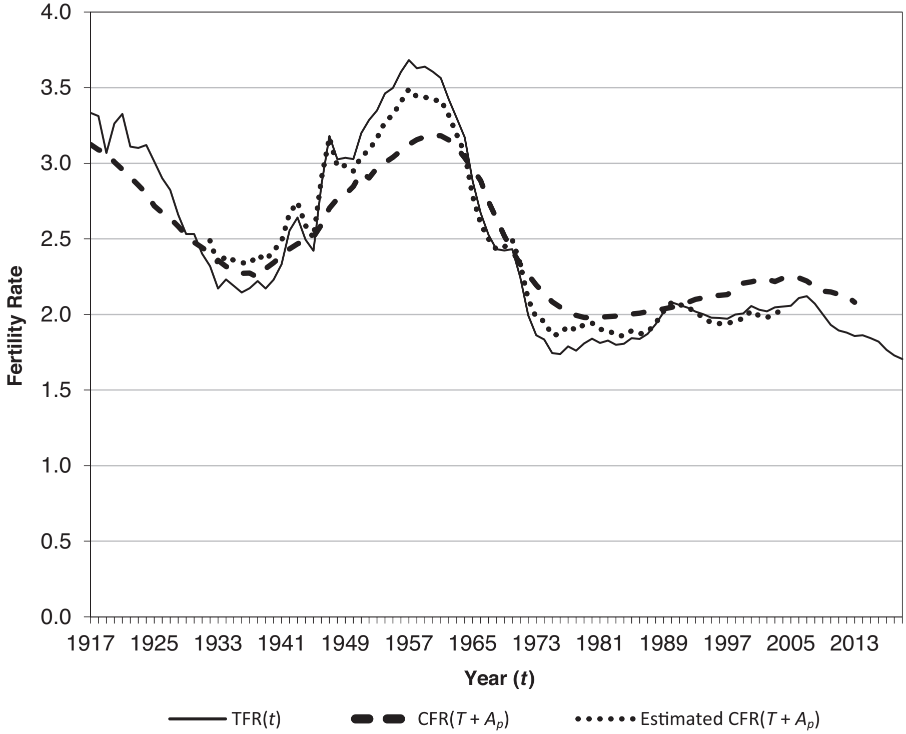

Fig. 3 Observed period and cohort total fertility and estimated total fertility, United States, 1917–2019

substantial increase in A_p, and relatedly in the fertility rates at higher ages, directly counters the model assumption of constant proportional fertility and provides a stringent test of the robustness of the model.

The estimated CFR trajectory in Figure 3 was calculated under the cubic assumption using Eq. (15). For each year t, a cubic curve was fit to TFR values for years $t-15$, $t-5$, $t+5$, and $t+15$, and the parameters of that curve were used in Eq. (15).

The 30-year period was chosen as it approximates both the length of a generation and the reproductive age span. Alternative specifications were considered, but differences were modest. The details are given in section G of the online appendix, and alternative estimates are shown in Appendix Table 1.

Figure 3 shows that the cubic estimated CFR tracks the observed CFR quite well, although it is a bit closer to the corresponding TFR. The largest errors are during the baby boom of the 1950s and the birth dearth of the 1970s, where the estimated CFR still outperforms the TFR. The robustness of the cubic CFR values to violations of the constant proportional fertility assumption is quite striking. Appendix Tables 1 and 2 show year-by-year and overall differences between the observed CFR and the different estimates. The linear or lagged TFR estimate was off by 0.196. On a base of 2, that is a percentage error of 9.8%. The average difference from the Eq. (15) cubic estimates was between 0.144 and 0.157, giving percentage errors of 7.2% to 7.8%.

Equations (14) and (15) relate period total fertility to cohort total fertility, but those relationships only partially constrain *age-specific* fertility, allowing many possible fertility schedules. One reasonable way to calculate age-specific rates is by a normal curve parameterization, which only requires knowledge of the mean and variance and is fully consistent with the derivation of Eq. (15). Approximating age-specific fertility by a normal curve ignores the typical right-handed skew of the fertility rates but is a frequent practice in demography (*cf.* Keyfitz 1977: chapter 6).

Figure 4 shows observed age-specific fertility and the fertility rates estimated from a normal curve parameterization for the United States for 1925, 1950, 1975, and 2000. (Details of the method used are in section I of the online appendix.) The fit of the estimates is quite good for 1975, and so-so for the other three years, where the skew in the observed rates is more pronounced. Thus plausible age-specific fertility rates can be estimated, even though the relationships derived here between the TFRs and the CFRs only loosely constrain the age curve of fertility.

To sum up, the model-based relationships of Table 1 perform reasonably well, even though the constant fertility proportions assumption is substantially violated. Alternative choices in operationalizing the model make little difference. Period fertility, especially when adjusted, consistently approximates the cohort fertility of U.S. women born 26–29 years earlier.

Forward- and Back-Projecting Cohort Total Fertility

Now let us consider whether the relationships explored here can extend the CFR series, first further into the future and then to earlier cohorts. With A_p (2019) equal to 29.62 years, TFR(2019) corresponds to the birth cohort of 1989–1990, while Table 3 only provides CFRs through the birth cohort of 1984.

One way to further extend the CFR series is to complete the experience of all active cohorts. The study of Bohk-Ewald et al. (2018) indicates that simply assuming that the age-specific birth rates of the latest year continue to apply generally yields quite good results. Table 3 uses 2019 experience in that way but, to minimize error, only at ages above 35. Those ages account for only 19% of 2019 fertility, while ages 30 and above account for 48% of 2019 fertility. Thus the assumption that fertility

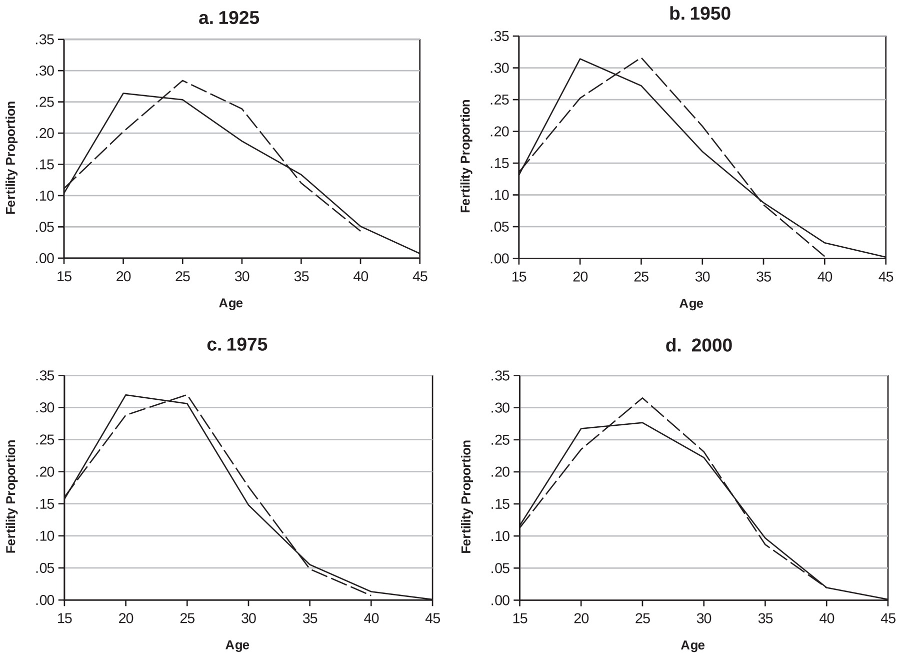

Fig. 4 Age-specific fertility proportions for the United States in 1925, 1950, 1975, and 2000, both observed (solid lines) and implied by observed fertility means and variances under a normal curve parameterization (dashed lines)

above age 30 remains constant at 2019 levels is much stronger than the one used to calculate CFR values in Table 3.

A second way to extend the series is by the linear assumption of Eq. (5), which takes TFR(2019) alone as the estimate of CFR(1989–1990). From the relationships in Figure 3, that assumption is a bit crude.

A third possibility is to use the cubic trajectory assumption of Eq. (14) and estimate CFRs up to the cohort of 1989–1990 using Eq. (15). That way appears promising, but implementing it requires fitting cubic curves to determine parameters b and d. Formally, that can be done by using the last 30 years of TFR data, as described in section H of the online appendix. Substantively, however, there is no escaping the use of a cubic curve that extends the fertility series to 2034–2035, some 15 years beyond the last data point. Doing so places extraordinary reliance on the cubic assumption.

Table 4 presents results from those three approaches for the years 2005–2019. Column 4—the Observed TFR based on fertility to at least age 35—gives the most reliable figures, but only goes to the cohort born in 1984. Comparing the figures in column 4 to the corresponding TFR indicates that the observed CFRs are from 0.10 to 0.24 higher (see column 5).

Column 6 shows that the cubic projected CFRs increase to a maximum of 2.076 for the cohort of 1979, and then decline steadily to 1.599 for the cohort of 1989–1990. For the years 2005–2013, column 7 shows how those projected CFRs relate to the

Table 4 Cubic and other projections of cohort fertility for U.S. cohorts born in 1977 through 1989–1990

Year (*t*) (1)	Cohort (*T*) (2)	Observed TFR(*t*) (3)	Observed CFR(*T*) (4)	Observed CFR(T)−TFR(*t*) (5)	Cubic Projected CFR(*T*) (6)	Observed−Projected CFR(*T*) (7)	TFR(*t*)−Projected CFR(*T*) (8)
2005	1977	2.057	2.234	0.177	2.024	0.216	0.033
2006	1978	2.108	2.243	0.135	2.070	0.173	0.038
2007	1979	2.120	2.221	0.101	2.076	0.145	0.044
2008	1980	2.072	2.183	0.111	2.022	0.161	0.050
2009	1981	2.002	2.154	0.152	1.946	0.208	0.056
2010	1981–1982	1.931	2.150	0.219	1.870	0.280	0.061
2011	1982–1983	1.894	2.134	0.240	1.828	0.305	0.066
2012	1983	1.880	2.122	0.242	1.809	0.313	0.071
2013	1984	1.858	2.081	0.223	1.782	0.299	0.076
2014	1985	1.862			1.782		0.080
2015	1986	1.844			1.758		0.086
2016	1987	1.820			1.730		0.090
2017	1987–1988	1.766			1.669		0.097
2018	1988–1989	1.730			1.629		0.101
2019	1989–1990	1.705			1.599		0.106

Source: See text and Section H in the online appendix for further details.

observed CFRs in column 4. The cubic projected CFRs are uniformly higher, by 0.14 to 0.31, and for the last three years of the comparison are 0.3 larger. That suggests that the cubic curve may be falling too rapidly.

In sum, while each has its limitations, the three approaches can offer a plausible range for CFRs through the birth cohort of 1989–1990. Cohort fertility is not likely to fall below 1.6, the cubic projection figure. Taking the 2019 TFR, the 1989–1990 cohort will have a CFR of 1.7. However, the TFR series is about 0.2 below the observed series in column 3, so completed family size may be as high as 1.9. In any case, within the likely range of 1.6 to 1.9, the CFR for the 1989–1990 cohort will clearly be below replacement and very likely the lowest CFR in American history.

Similar issues arise in back projections to estimate the fertility of earlier cohorts (see section H of the online appendix for details). To extend the cubic approximation back to cohorts born in 1888–1889, the cubic curve based on the years 1917, 1927, 1937, and 1947 has to be extrapolated back to 1887. The results are shown in Table 5. The cubic back-projected CFRs are given in column 5 and show a largely steady decline from 4.137 for the 1888–1889 cohort to 2.614 for the 1903 cohort. The differences from the CFRs reported in Heuser (1976), shown in column 6, reveal that the back-projected figures are uniformly larger, with differences greater than 0.7 for the cohorts born from 1888–1889 through 1898. That very poor performance by the back projection reflects the dangers involved in extrapolating cubic curves. In contrast, column 7 shows that the lagged TFR values perform fairly well, with an average error of 0.18.

Table 5 Linear and cubic back projections of cohort fertility for U.S. cohorts born in 1888–1889 through 1903

Year (t) (1)	Observed TFR(t) (2)	Cohort (T) (3)	Observed CFR(T) (4)	Cubic Back-Projected CFR(T) (5)	Projected−Observed CFR(T) (6)	TFR(t)−Observed CFR(T) (7)
1917	3.333	1888–1889	3.125	4.137	1.012	0.208
1918	3.312	1889–1890	3.090	4.055	0.965	0.222
1919	3.068	1890	3.072	3.788	0.716	−0.004
1920	3.263	1891–1892	3.008	3.941	0.933	0.255
1921	3.326	1892–1893	2.960	3.962	1.002	0.366
1922	3.109	1893–1894	2.909	3.704	0.795	0.200
1923	3.101	1894–1895	2.856	3.654	0.798	0.245
1924	3.121	1895–1896	2.802	3.632	0.830	0.319
1925	3.012	1897	2.718	3.481	0.763	0.294
1926	2.901	1898	2.672	3.328	0.656	0.229
1927	2.824	1899	2.635	3.207	0.572	0.189
1928	2.660	1900	2.583	2.998	0.415	0.077
1929	2.532	1901	2.525	2.826	0.301	0.007
1930	2.532	1902	2.477	2.786	0.309	0.055
1931	2.402	1903	2.442	2.614	0.172	−0.040

Source: See text and Section H in the online appendix for further details.

Summary and Conclusions

The relationship between period and cohort fertility has been a focus of demographic interest since Ryder's pioneering 1964 paper. Here, a new approach to relating those two perspectives is developed, based on the assumption of constant age-specific fertility proportions. When the level of fertility follows a linear, quadratic, or cubic trajectory, explicit relationships, shown in Table 1, are found between period total fertility at time t and the total fertility of the cohort born at time T, where $T=t-\mu$ and μ is the constant period mean age of fertility. Analogous relationships are found for the period and cohort mean ages of fertility, and a simple relationship unites both period and cohort total fertility with the average fertility of a year's actively childbearing cohorts.

These new relationships between period and cohort total fertility are quite robust to departures from the assumed pattern of constant age-specific fertility proportions. That is evident when estimated cohort total fertility is compared to observed cohort total fertility for the United States in 1917–2019. U.S. fertility was at an all-time low of 1.705 in 2019, and the data show substantial departures from the assumed fixed proportional fertility. Nonetheless, the theoretically derived estimates had errors averaging only 7–8%.

Both period and cohort perspectives illuminate fertility behavior. Period total fertility is not a "distortion" of cohort total fertility, but another manifestation of the same array of age- and time-specific fertility rates. As women bear children over their life course in the light of their previous fertility, the heuristic appeal of the cohort concept is undeniable. Nonetheless, people live year by year, and those period conditions influence both short-term and long-term behavior. What emerges from the present analysis is a fuller appreciation of the close relationship between period and cohort fertility. The two perspectives are complementary, and both contribute to our understanding of population fertility. ■

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Robert Schoen
rschoen309@att.net

Population Research Institute, Pennsylvania State University, University Park, PA, USA; https://orcid.org/0000-0001-6889-9954