The Components of Change in Population Growth Rates

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ABSTRACT The demographic balance equation relates the population growth rate with crude rates of fertility, mortality, and net migration. All these rates refer to changes occurring between two time points, say, t and $t + h$. However, this fundamental balance equation overlooks the contribution of historical fertility, mortality, and migration in explaining these population counts. Because of this, the balance equation only partially explains a change in growth rate between time t and $t + h$ as it does not include the contribution of historical population trends in shaping the population at time t. The overall population growth rate can also be expressed as the weighted average of age-specific growth rates. In this article, we develop a method to decompose the historical drivers of current population growth by recursively employing the variable-r method on the population's average age-specific growth rates. We illustrate our method by identifying the unique contributions of survival progress, migration change, and fertility decline for current population growth in Denmark, England and Wales, France, and the United States. Our results show that survival progress is mainly having an effect on population growth at older ages, although accounting for indirect historical effects illuminates additional contributions at younger ages. Migration is particularly important in Denmark and England and Wales. Finally, we find that across all populations studied, historical fertility decline plays the largest role in shaping recent reductions in population growth rates.

KEYWORDS Variable-r • Population growth • Population dynamics • Decomposition

Introduction

Over the past 50 years, the global population growth rate has declined by more than half, and current projections estimate that by 2100 the total world population could stabilize between 9 and 11 billion (United Nations 2019; Vollset et al. 2020). At the country level, the basic demographic balance equation implies that the population growth rate in a given period equals the crude birth rate minus the crude death rate plus the net migration rate observed over the time interval (Preston et al. 2000). However, the growth rate of the total population is an age-aggregated measure—that is, it hides the diversity in growth experienced across different age-groups. For policy-making and

planning purposes, understanding which segments of a population are growing rapidly and which are stagnating and declining is key for making informed decisions. An alternative to the balance equation is to calculate the population growth rate as the weighted average of the age-specific growth rates (Horiuchi 1991).

Both of these approaches to estimating a population's growth rate (the balance equation and average age-specific growth rates) correspond to a "current rates" perspective, considering changes in the population observed over a specific period in time, say, between times t and $t + h$. However, the "current rates" perspectives hides the historical contributions of fertility, mortality, and migration in shaping the current population. Declines in fertility and mortality during the course of the demographic transition cause important changes in a population's age composition (Bongaarts 2009). Likewise, the structure of the population influences growth. For example, an aging society might experience more deaths at older ages where population counts are greater and where death rates are the highest. Such a population may also experience lower fertility owing to smaller numbers of individuals of reproductive age, and could attract more young immigrants to replace their lost workforce. In contrast, a young society with a large proportion of individuals of reproductive age is prone to experiencing high rates of both fertility and migration (Bernard et al. 2014).

Population policy based on a current rates perspective might result in distorted outcomes since it takes the population counts under study as given, overlooking the demographic history that shaped them. The variable-r approach presents a method to overcome this shortcoming, bringing the desired historical information into perspective (Arthur and Vaupel 1984; Preston and Coale 1982). However, the use of the variable-r method also comes with caveats, since the fertility, mortality, and migration components are not fully disentangled by simply substituting the original equations (Lee and Zhou 2017; Murphy 2017; Preston and Stokes 2012). More specifically, the fertility component is not purely fertility since it includes elements of the population structure that all three elements (past fertility, mortality, and migration) have influenced.

In this article, we propose a procedure that fully separates the past contribution of each of these demographic components through a recursive application of the variable-r method. The variable-r method relates age-specific growth rates at a given age x and time t to the probabilities of surviving and migration from birth to age x, and the growth rate of those births at time $t - x$. This latter component allows for further disentangling the effect of fertility from the "indirect effect" of mortality, migration, and fertility of past generations. Previous efforts of separating the fertility component from age-specific growth rates have used aggregated measures of fertility such as the total fertility rate and gross reproduction rate (Lee and Zhou 2017; Murphy 2017; Preston and Vierboom 2021). We propose an alternative approach, centered not on aggregated measures but on information on the distribution of births over ages of mothers and the age-specific fertility rates at a given time. Thus, our contribution is to derive a direct disaggregation of the population growth rate at each age into components of fertility, mortality, and migration, and of the fertility, mortality, and migration patterns experienced by the historical cohorts that produced today's population. We illustrate our method by identifying the unique contributions of survival progress, migration change, and fertility decline for current population growth in Denmark, England and Wales, France, and the United States.

Data

Our analyses use data from the Human Mortality Database (HMD) (n.d.) and the Human Fertility Database (HFD) (n.d.). The HMD and HFD contain high-quality historical data combining vital statistics and census counts or official population estimates, and standard methods are applied across all populations over time to ensure comparability (Jasilioniene et al. 2015; Wilmoth et al. 2017). Age-specific counts of population and death rates were obtained from the HMD, and age-specific counts of births and fertility rates were obtained from the HFD. Here we present the results for four selected female populations from Denmark, England and Wales, France, and the United States. The first three populations have comprehensive demographic data from the nineteenth, twentieth, and twenty-first centuries. The distribution of births over the ages of mothers first observed in the series of HFD was fixed for earlier periods when information was not available (a sensitivity analysis of this assumption is presented in the online Appendix 1). For the United States, information from the HMD and HFD allows us to build a truncated historical series from 1933 to 2018. To complete the U.S. series from 1908 to 1932, we rely on birth data from the Centers for Disease Control and Prevention (CDC 2020), mortality data from the Social Security Administration (Bell and Miller 2005), and population estimates from the U.S. Census (U.S. Census Bureau 2016). More detail on this process is found in online Appendix 2. This additional information is required to explore the drivers of growth rates in the United States at ages 75 and above.

Methods

Horiuchi (1991) introduced relations between the growth rate of the population and the age-specific measures of fertility, mortality, and migration. In this section, we present these methods and extend them to include new terms with historical data (see online Appendices 3–5 for details of these derivations). To do so, we use the variable-r method (Arthur and Vaupel 1984; Preston and Coale 1982).

Components of Population Growth

Let a dot on top of a variable denote the derivative of a function with respect to time (Newton's dot notation for derivatives (Newton 1704)). For example, the population growth rate is the relative change in population counts and is calculated as the relative derivative with respect to time or $r(t) = \dfrac{\dot{N}(t)}{N(t)}$, where $N(t)$ denotes the population count at time t. Since the total population count at time t is an age-aggregated value of age-specific population counts, or $N(t) = \int_0^\omega N_x(t)dx$, then the population growth rate can be rewritten as a weighted average,

$$r(t) = \bar{r} = \int_0^\omega r_x(t)c_x(t)dx, \tag{1}$$

where the bar on top of the variable $\bar{r}$ indicates the weighted average growth rate, and $r_x(t) = \dfrac{\dot{N}_x(t)}{N_x(t)}$ and $c_x(t) = \dfrac{N_x(t)}{N(t)}$ are the age-specific growth rates and population composition at age x and time t, respectively (Horiuchi 1991).

The age-specific growth rates in Eq. (1) can be further decomposed to include information on growth rate at birth and the change over time in the probability of survival and migration between birth and age x (Horiuchi and Preston 1988) as

$$r_x(t) = r_B(t-x) + \Delta S^{t-x} + \Delta M^{t-x}, \tag{2}$$

where $r_B(t-x)$ denotes the growth rate at birth and $\Delta S^{t-x} = \dfrac{\dot{S}^{t-x}(x)}{S^{t-x}(x)}$ and $\Delta M^{t-x} = \dfrac{\dot{M}^{t-x}(x)}{M^{t-x}(x)}$ are the terms that compare the change in probabilities of surviving and net migration for the birth cohort $t - x$. According to Preston and Stokes (2012), the net migration effect can be calculated as the residual term of the other three components in Eq. (2), as opposed to including country-specific data on actual migration flows. The same procedure is followed here.

Substituting Eq. (2) in Eq. (1) returns

$$\bar{r} = \overline{r_B} + \overline{\Delta S} + \overline{\Delta M}, \tag{3}$$

where the bars on top of the variables $\overline{r_B}$, $\overline{\Delta S}$, and $\overline{\Delta M}$ indicate the weighted average of the growth rate at birth and of changes in survival and migration, respectively. As in Eq. (1), the weighting function is the structure of the population or proportion of individuals at each age with respect to the total population, or $c_x(t)$. Equation (3) corresponds to the decomposition of the growth rate for the population present at time t into their past survival, migration, and growth rate at birth.

Preston and Stokes (2012) referred to the growth rate at birth as the fertility component of the age-specific growth rates. However, as pointed out by Lee and Zhou (2017), the change in cohort birth counts, captured by the growth rate at birth, is not purely a fertility component since it can also be decomposed into fertility and population structure components. Murphy (2017) disentangled those birth counts as the product of age-aggregated fertility and population at risk terms.

In contrast, our proposed method focuses on age-specific components of fertility and population counts. The total birth count is the product of the fertility rates $f_a(t)$ and the population counts $N_a(t)$ over the reproductive age limits α and β, as $B(t) = \int_\alpha^\beta f_a(t) N_a(t) da$. We further disentangle the growth rate at birth into two components as

$$r_B(t-x) = \int_\alpha^\beta \frac{\dot{f}_a(t-x)}{f_a(t-x)} c_B(a, t-x) da + \int_\alpha^\beta r_a(t-x) c_B(a, t-x) da, \tag{4}$$

where $c_B(a, t-x) = \dfrac{B_a(t-x)}{B(t-x)}$ is the composition of births born to women age a over all births at time $t - x$. The first component in Eq. (4) is purely the contribution of fertility change, while the second is the change in the population at risk, or age-specific growth rates of women at reproductive ages. We will refer to this latter new set of growth rates as the growth rates of mothers.

Further substituting Eq. (4) into Eq. (3), we obtain

$$r(t) = \overline{\Delta S} + \overline{\Delta M} + \widetilde{\Delta f} + \tilde{r}, \tag{5}$$

where the tilde averages correspond to fertility change and population growth of mothers weighted by both the population and birth composition, or $c_x(t)$ and $c_B(a, t-x)$.

Equation (5) differs from Eq. (3) since the fertility and additional growth rate in the former equation correspond to the reproductive age-groups, that is, mothers that conceived the babies at time $t - x$, which later became the individuals present at time t. Thus, this approach isolates the direct effect of fertility on population growth at time t, independent of the age-specific growth rates of women of reproductive age. The fertility term includes the demographic contribution of mothers at different times, starting at time t and reaching back to the year when the oldest individuals present in the population at time t were born.

The last average of age-specific population growth in Eq. (5) corresponds to the growth of the group of women of reproductive age who conceived the births of the original growth rate at birth, $\overline{r_B}$ in Eq. (3). This "growth rates of mothers" can be further disentangled into their growth rate at birth, survival, and migration using Eq. (2), which further changes Eq. (5) into

$$r(t) = \overline{\Delta S} + \overline{\Delta M} + \widetilde{\Delta f} + \widetilde{\Delta S} + \widetilde{\Delta M} + \widetilde{r}_B, \tag{6}$$

with the tilde on top of the variables corresponding to the $c_x(t)$ and $c_B(a, t-x)$ weighting as in Eq. (5). We then further decompose the growth rate at birth of the mothers ($\widetilde{r}_B$), using Eq. (4) to disentangle the roles of fertility and population growth (in this case, population growth of grandmothers of the current population at time t), so that Eq. (6) becomes

$$r(t) = \overline{\Delta S} + \overline{\Delta M} + \widetilde{\Delta f} + \widetilde{\Delta S} + \widetilde{\Delta M} + \widehat{\Delta f} + \hat{r}, \tag{7}$$

with the hat on top of the variable corresponding to the fertility and growth rate of those grandmothers. The weights in these hat-averages are the distribution of births when the mothers were born, or $c_B(y, t-x-a)$, and when each of the cohorts present at time t were born, $c_B(a, t-x)$, as well as the population composition at time t, or $c_x(t)$. The online Appendices 2–3 include more detailed derivations of the equations as well as visual representation in the lexis diagram of the elements included in Eqs. (1), (3), (5), and (6).

This recursive process could be repeated again, further separating population growth $\hat{r}$ by including a component for births from an additional previous generation (i.e., the great-grandparents of the current population). However, after the iterations of the terms of $\overline{r}$ in Eq. (3), $\widetilde{r}$ in Eq. (5), and $\hat{r}$ in Eq. (7), the fertility and growth rates correspond almost entirely to cohorts no longer present in the population at time t. Furthermore, as proposed by ergodicity theorems, the demographic past is progressively reshaped by vital events and eventually it ceases to determine the future course of the structure of the population (Arthur 1982). Thus, it is possible to truncate the calculations at this stage, indicating the remaining $\hat{r}$ as the residual term.

The direct and indirect effects of fertility, mortality, and migration described in the preceding can all be applied to the population of females, where the fertility component in Eq. (4) includes only female births. However, although it is possible to obtain the survival, migration, and growth at birth elements as in Eq. (3) for males, it is not possible to obtain the contribution of fathers or grandfathers, since only fertility information for women is available. Instead, Eq. (4) is adapted to identify the fertility

of the mothers that conceived those baby boys. The online Appendix 5 compares the results for females with those for males, highlighting the similarity given the same set of fertility and growth rate elements in Eqs. (4)–(7).

The following section presents four illustrative cases of populations with diverse trajectories of the components of population growth: Denmark, England and Wales, France, and the United States. Results for a wider number of countries are included in the interactive Shiny app (https://demo268.shinyapps.io/Historical/). All the calculations in this study were carried out using R software (R Core Team 2021), and commented R code is included in the repository https://github.com/tyaSHEN/histdemo.

Results

This section follows the same structure as the foregoing methods section, focusing in turn on: (1) findings for Eq. (3) separating population growth into survival, migration, and growth at birth; (2) further disaggregating the growth at birth into fertility and population growth components as in Eq. (5); and (3) decomposing this population growth component further into the historical contributions of fertility, survival, and migration to population growth as in Eq. (7).

The female populations in Denmark, England and Wales, France, and the United States grew at rates of 0.50%, 0.69%, 0.42%, and 0.73% from 2008 to 2018. Table 1 presents the historical components of growth rate (mortality, migration, and growth rate at birth multiplied by 100) as presented in Eq. (3) for these four populations. Considering the components of the variable-r model in Eq. (3), progress in survival from comparing changes in survival ($\overline{\Delta S}$) of successive cohorts shows a narrow range of contributions from 0.21 to 0.35 on average per year. In other words, mortality improvements are observed in all countries, and contribute positively to the population growth rate. However, if only mortality improvements had changed during the analyzed time, all growth rates would have been smaller, with the extreme case observed in the United States with only a third of the observed growth (0.21% vs. 0.73%) resulting from mortality improvements. The other two components are more variable across populations, with the migration component ranging from −0.09 to 0.43 and the growth rate at birth contributing between −0.24 and 0.38. Migration was the main contributing factor to continued positive population growth in Denmark and England and Wales, but in France migration actually acted to reduce the population over this period (−0.09). France has experienced fluctuations in migration counts over the twentieth century (Ogden and White 2002), but it is less clear how this translates to cohorts, as captured in our equations. Two factors can help explain the values seen for France. First, over a cohort perspective, return migration cancels out, for example, if individuals emigrate at a young age and later in life immigrate back. Second, if consecutive cohorts are migrating at the same intensity, then the change in the migration component will be zero. The final component, the growth rate at birth, has a pronounced negative effect on Danish population growth (−0.24), but a strong positive effect in the United States (0.38).

In Figure 1, the age-specific growth rates, as well as the contribution of each of the different components of the population growth as in Eq. (3), are explored. These age-specific contributions include the age composition of the population ($c_x(t)$) as the

Table 1 Female population growth rate and its historical components: Mortality, migration, and growth rate at birth (Eq. (3)), for selected countries from 2008 to 2018

		Denmark	England and Wales	France	United States
$\bar{r}$	Growth rate	0.50	0.69	0.42	0.73
$\overline{\Delta S}$	Mortality	0.31	0.32	0.35	0.21
$\overline{\Delta M}$	Migration	0.43	0.38	−0.09	0.14
$\overline{r_B}$	Growth rate at birth	−0.24	−0.01	0.16	0.38

Source: Author's calculations based on the HMD (n.d.) and HFD (n.d.) for historical components from 1853 to 2018, and 1933 to 2018 for the United States. Additionally, the U.S. data were extended to the years 1908 to 1932 with information from the CDC (2020), the Social Security Administration (Bell and Miller 2005), and U.S. Census Bureau (2016), as indicated in the online Appendix 2.

weighting function as in Eq. (1), so that the area under each line is equal to the overall contribution of that component to aggregate population growth. We find that the cohort-specific survival improvements are positive for every population, with these improvements having the largest influence at older ages (although contributions of improved survival to overall growth are somewhat smaller in the United States than in other countries). At those older ages, specifically 90 and above, the survival component outweighs all other components. The age-specific contribution of migration tends to move in opposition to the age-specific growth rate (i.e., positive when the other is negative) and has an especially pronounced contribution in Denmark and England and Wales for age-groups from the late 30s to the late 50s. Finally, the overall pattern of age-specific growth rates is closely aligned with the growth rate at birth. This concurrence highlights the relevance of this component in explaining fluctuations in the overall age-specific growth rates, from the high growth rate at birth for the baby boomers (in Figure 1 shown in their 70s) to the booms and busts in fertility that have followed.

Table 2 presents a further decomposition of the two components of the growth rate at birth—into fertility ($\widetilde{\Delta f}$) and the growth rate of mothers ($\tilde{r}$), as shown in Eq. (4)—for the four selected countries. All four countries have experienced reductions in fertility, although this decline is particularly strong in Denmark (−0.47) and the United States (−0.53). As in Table 1 for mortality, the scenario where only fertility declines results in a decline in total growth rates in all four populations between 2008 and 2018. However, the high growth rate of women of childbearing ages more than compensates for the decline in fertility rates in the United States (0.91 vs. −0.53) and France (0.34 vs. −0.18), counterbalances it in England and Wales (0.20 vs. −0.21), but is less than half of the decline in fertility in Denmark (0.22 vs. −0.47).

Figure 2 presents the results from Table 2 in age-specific fashion, plotting the age-specific contributions of fertility and the growth rate of mothers on the growth rate at birth. Across the four countries, we find strong opposing trends between fertility decline and the growth rates of mothers for cohorts born after the baby boom (those aged in their 30s to 50s in 2018). Briefly interpreted, these findings highlight that the relatively small changes in aggregate growth rate in these ages hide the large, offsetting influences of the post–baby boom decline in fertility combined
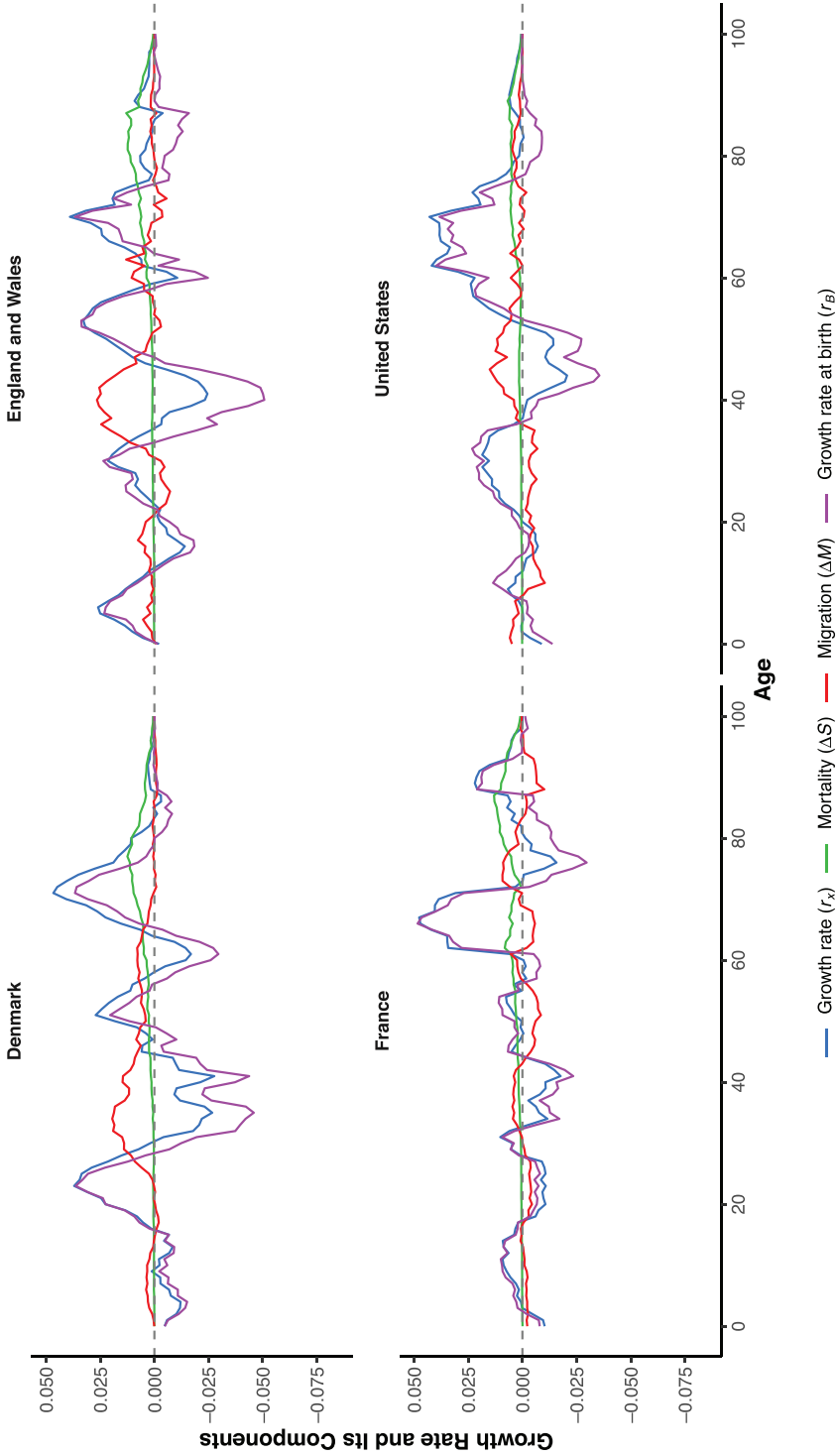
Fig. 1 Age-specific components of female population growth rate: Mortality, migration, and growth rate at birth (Eq. (3)), for selected countries from 2008 to 2018. *Source:* See Table 1.

Table 2 Female population growth rate, and growth rate at birth and its historical components: Fertility and growth rate (Eqs. (3) and (5)), for selected countries from 2008 to 2018

		Denmark	England and Wales	France	United States
$\bar{r}$	Growth rate	0.50	0.69	0.42	0.73
$\overline{r_B}$	Growth rate at birth	−0.24	−0.01	0.16	0.38
$\widetilde{\Delta f}$	Fertility	−0.47	−0.21	−0.18	−0.53
$\tilde{r}$	Growth rate of mothers	0.22	0.20	0.34	0.91

Source: See Table 1.

with the large cohorts of women produced during the baby boom. These patterns are most pronounced in the United States and France, and are somewhat smaller in Denmark and England and Wales, where the growth rate of mothers played a less substantial role.

Following on these results, Table 3 presents a disaggregation of the components of the population growth rate as in Eq. (7): decomposing the growth rates of mothers into components for their survival, migration, and growth rate at birth, and further decomposing this "growth rate at birth of mothers" into components for the fertility of their grandmothers and their growth. The limited time series of data from the United States means that these additional decompositions from Eq. (7) are not possible; results for the United States present the combined results of Tables 1 and 2 as in Eq. (5). Broadly, we find that mortality change is acting to increase the growth rate, fertility change is acting to reduce the overall growth rate, and migration's contributions are more mixed. Survival improvements highlight continued progress with respect to historical populations over a century ago, demonstrating an accumulation of survival improvements from cohort to cohort. As Figure 1 illustrates, these improvements in survival are particularly important in increasing the growth rate at older ages. The contribution of changes in mortality to population growth is substantial in France, Denmark, and England and Wales. This mortality contribution is smaller in the United States, although these figures do not account for the additional generations included in Eqs. (6) and (7). In Denmark, England and Wales, and France, the negative contribution of fertility includes the effect of several generations (mothers and grandmothers of the current population). We find that fertility has a particularly large influence on population growth in Denmark (−1.48 for fertility vs. 1.18 for mortality and migration combined), and its influence is considerable in France (−0.59 vs. 0.61) and England and Wales (−0.83 vs. 1.21). As opposed to the scenarios where only mortality improvements occur (described for Table 1) or where only declines in fertility are present (described for Table 2), no similar partition can be done for the values in Table 3. The interconnected relations between the different demographic components (fertility, mortality, and migration) over generations only allow the decomposition of their contributions to the total growth rate between 2008 and 2018.

Figure 3 includes the age disaggregation of the components in Table 3. For the United States, this figure represents a combination of the information in Figures 1 and 2, while for the rest of the countries these results include contributions to changes in
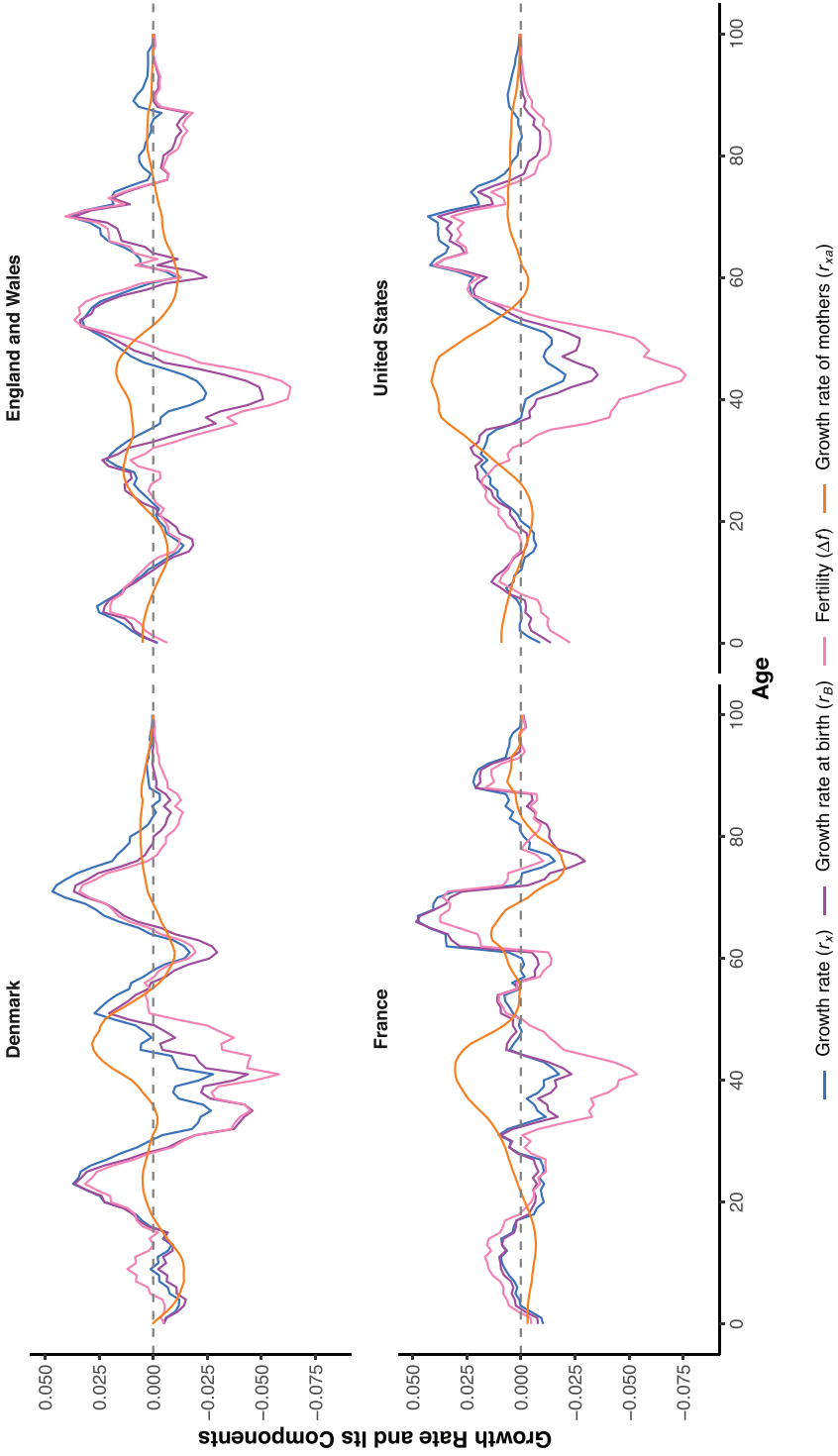
Fig. 2 Age-specific components of female growth rate at birth: Fertility and growth rate (Eqs. (3) and (5)), for selected countries from 2008 to 2018. *Source*: See Table 1.

Table 3 Female population growth rate and its historical components: Fertility, mortality, migration, and residual growth rate (Eq. (7)), for selected countries from 2008 to 2018

		Denmark	England and Wales	France	United States
$\bar{r}$	Growth rate	0.50	0.69	0.42	0.73
$\overline{\Delta S} + \widetilde{\Delta S}$	Mortality	0.56	0.61	0.71	0.21
$\widetilde{\Delta f} + \widetilde{\Delta f}$	Fertility	−1.48	−0.83	−0.59	−0.53
$\overline{\Delta M} + \widetilde{\Delta M}$	Migration	0.63	0.60	−0.09	0.14
$\hat{r}$	Growth rate of grandmothers	0.79	0.31	0.40	0.91

Note: For the United States, only data for single fertility, mortality, and migration components are included as in Eq. (5).

Source: See Table 1.

mortality, migration, and fertility from an additional previous generation (the grandmothers of today's population, as in Eq. (7)). While the age-specific growth rates remain fixed in Figures 1 to 3, all the other components include the contributions of historical demographic changes that have indirectly influenced the current age-specific rates of population growth. With this additional generation included, survival progress is now evident at younger ages, demonstrating the continued positive influence of historical mortality improvements on the growth rate of today's population. Migration plays a substantial role in increasing growth at younger ages in Denmark and England and Wales, and in all populations tends to be countercyclical with the contribution of fertility. Finally, this figure emphasizes the role of historical declines in fertility as a driver of current growth rates. The inclusion of an additional generation of women (mothers and grandmothers) leads to the fertility component's contribution shifting downward (when comparing Figures 1 and 3) in each population at every age.

Discussion

Our belly buttons are the mark of early life attachment to our mothers, who have the very same mark of their own early life connection to our grandmothers. Similarly, the prevailing age-specific growth rates experienced by today's population are an accumulation of a long history of the fertility, mortality, and migration of past generations. The fertility and mortality changes experienced during the demographic transition, as well as migration, have shaped the current population's age structure (Bongaarts 2009). Thus, to understand population growth today, it is vital to assess those historical changes from the past. In this article, we have demonstrated that it is possible to partition these currently existing rates into historical components representing the direct and indirect effects of fertility, mortality, and migration on today's growth.

The results of our analyses highlight that the growth of today's population represents the combined effects of decades of dynamic population change. These rates of age-specific growth are the result of large and often offsetting influences of historical patterns of mortality, fertility, and migration. The additional disaggregation presented
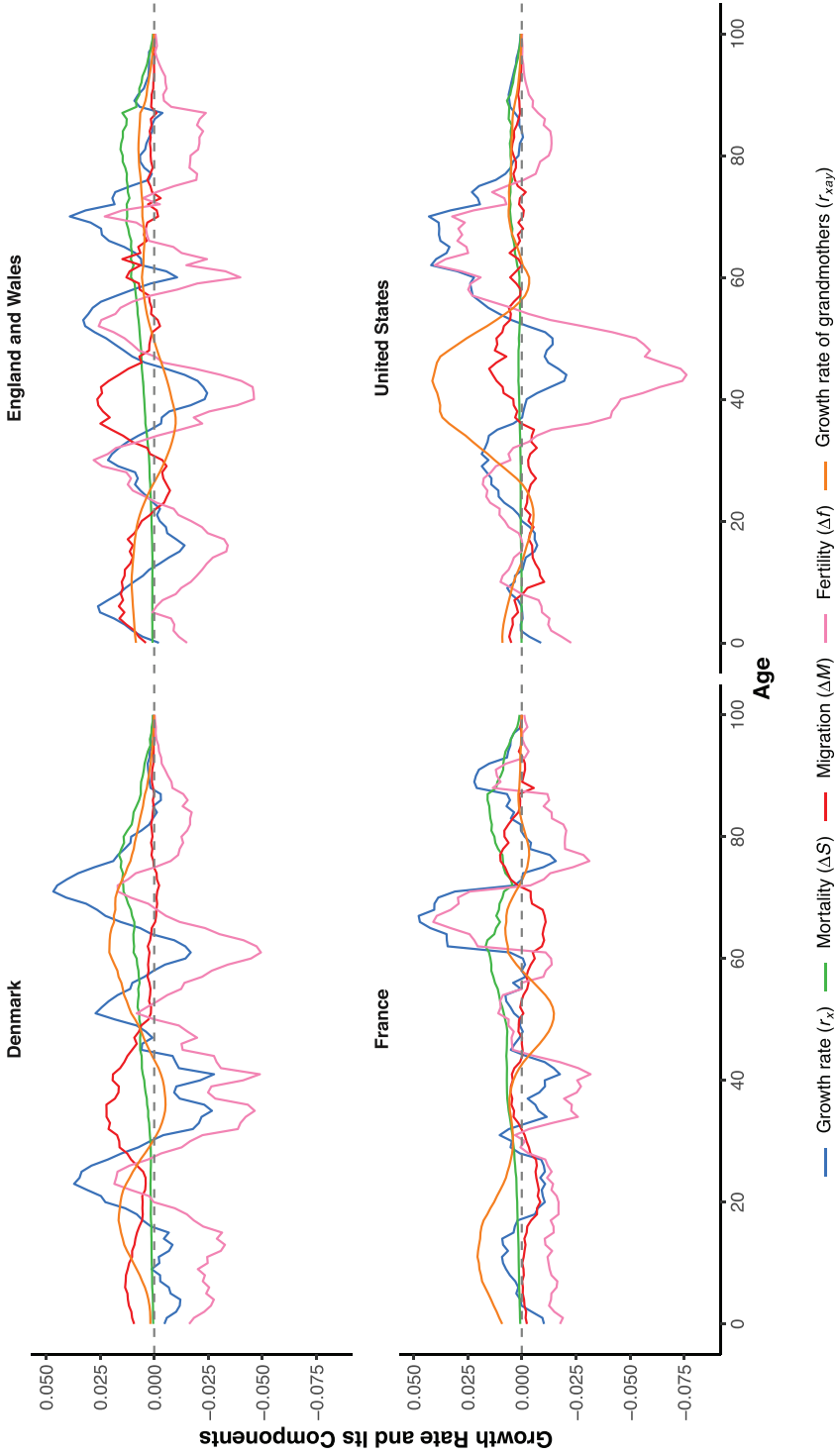
Fig. 3 Age-specific components of female growth rate at birth: Fertility, mortality, migration, and residual growth rate (Eqs. (5) and (7)), for selected countries from 2008 to 2018. *Source*: See Table 1.

here provides demographers with a clear window into the historical determinants shaping current population growth, allowing for new insights into the similarities, and differences, in the factors shaping growth across different populations.

Concerns about population growth are pervasive in policy discussions, from worries about rapid growth in less developed regions to apprehension about slowing (or negative) growth in many high-income contexts. Understanding the demographic processes driving current growth rates is essential for policymakers to make informed decisions about prospects for future national-level population growth and sustainable development. This article provides new insight into how historical patterns of fertility, mortality, and migration combine to produce currently observed rates of population growth and identifies the relative influence of each of these demographic factors on aggregate growth. Identifying these components of change sheds light on how the drivers of a population's growth are balanced across demographic components. Understanding whether current rates of growth are being driven by a balanced mix of fertility, mortality, and migration, or are resulting from an extreme, counterbalancing combination of these factors, has considerable implications for shaping population policies.

Our proposed decomposition of the population growth rate includes a residual term. The latter is not an error term, but rather represents the growth rate at reproductive ages for mothers (in the case of the United States) or for grandmothers (for the remaining countries in the analysis). Limitations on historical data availability, particularly on age-specific fertility rates, impede decomposition of these growth rates to further generations. However, including additional previous generations of data would be of limited use to our analyses. Our method focuses on decomposing the population growth rate between years t and $t+h$ as a function of the demographic events (mortality, fertility, and migration) that occur in the population present in years t and $t+h$. Thus, we stop our substitutions of the variable-r relations in Eq. (7), including information of mothers and grandmothers that were still present in the population between years t and $t + h$.

Studying the age-specific growth rates has further implications beyond the overall population growth rate studied here. Research on methods of decomposition has shown that changes over time in many demographic variables include age-specific population growth rates as key elements of their dynamics (Preston et al. 1989; Vaupel and Canudas-Romo 2003). For example, when looking at aging, Preston and Stokes (2012) reported changes in growth rates when assessing the change over time in the average age of the population, and also made comparisons between growth rates when assessing old-age dependency ratios. Even changes over time in the total fertility rate and life expectancy (which only depend on age-specific fertility and mortality rates) can be reexpressed as equations that include age-specific growth rates as elements of their dynamics (see the online Appendix 6). Hence, our results can be applied beyond the current examination of the overall population growth rate and adapted to any demographic measure in which changes over time are captured by a set of age-specific growth rates. ∎

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